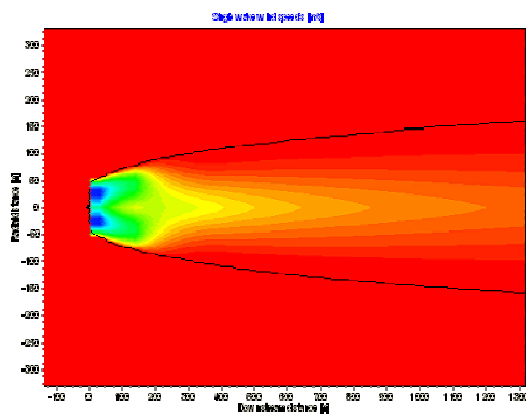
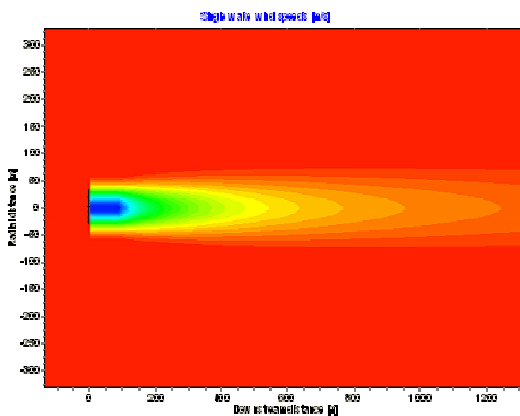
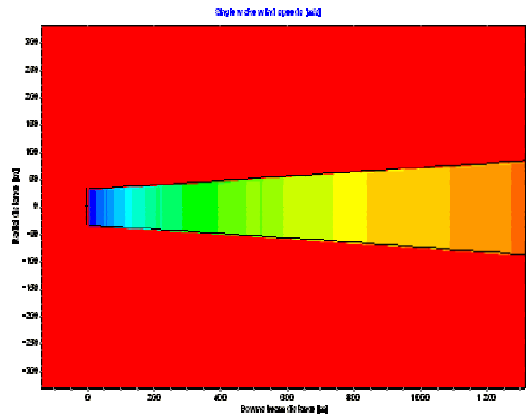


# WindPRO / PARK

## Introduction to Wind Turbine Wake Modelling and Wake Generated Turbulence



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**Front cover**

The front cover shows a wake development behind a single turbine. The wake velocities are calculated using the N.O. Jensen PARK model, the Ainslie model (eddy viscosity) and the G.C. Larsen model.

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## 1. Introduction to Wake Modelling

This paper gives a brief introduction to the concept of wake modelling for arrays of wind turbines. While WindPRO offers the opportunity to calculate the wind farm wakes with a number of different wake models, the user must choose and configure the model that fits the best into the area of application. This report gives selected background theory of single and multiple wake modelling and also on the implementation of the specific wake models. Descriptions of the different wake models are found in the succeeding chapters.



Figure 1: A cluster of four >2 MW semi-offshore turbines in Frederikshavn, Denmark.

### Introduction

When the turbine extracts power from the wind, a wake evolves downstream of the turbine. If another nearby turbine is operating within this wake, the power output for this downstream turbine is reduced when comparing to the turbine operating in the free wind. This reduction of power output is – on an annual basis – typically in the range of approximately 2% - 20%, depended on the wind distribution, the wind turbine characteristics and the wind farm (array) geometry.

The turbines operating in the wake are not only subjected to a decreased wind speed but also increased dynamic loading – arising from the increased turbulence induced by the upstream turbines. This increased turbulence must be accounted, when selecting a turbine suitable class of turbines. This is typically done though the specifications in the international codes – e.g. the IEC-61400-1 code for wind turbine structures.

The models available in WindPRO are currently all single wake models, i.e. models capable describing the flow downstream of one turbine. When having multiple turbines, the results from the single wake models are aggregated into a combined result by using empirical combination rules.

### The Wake after an Idealized Turbine

Assuming an idealized turbine – where flow around and behind the turbine is without rotation and friction - it is possible to derive some general and important equations describing the wake wind speeds. For further details please consult the publication by Andersen et al. [1]. The derivation is based on the simplified Bernoulli equation, stating that the mechanical energy per unit mass – along a streamline - is conserved:

$$\frac{\rho V^2}{2} + p = H \quad (1)$$

where  $\delta$  is the air density  
 $V$  is the wind speed  
 $p$  is the pressure  
 $H$  is the total energy (constant along any streamline)

## 1. Introduction to Wake Modelling

The Bernoulli equation gives the relation between pressure and wind speed, as the total pressure is constant along a streamline (streamline = a line which is drawn, such as it is always tangent to the velocity vector). Using the Bernoulli equation just before and after the rotor gives us two equations:

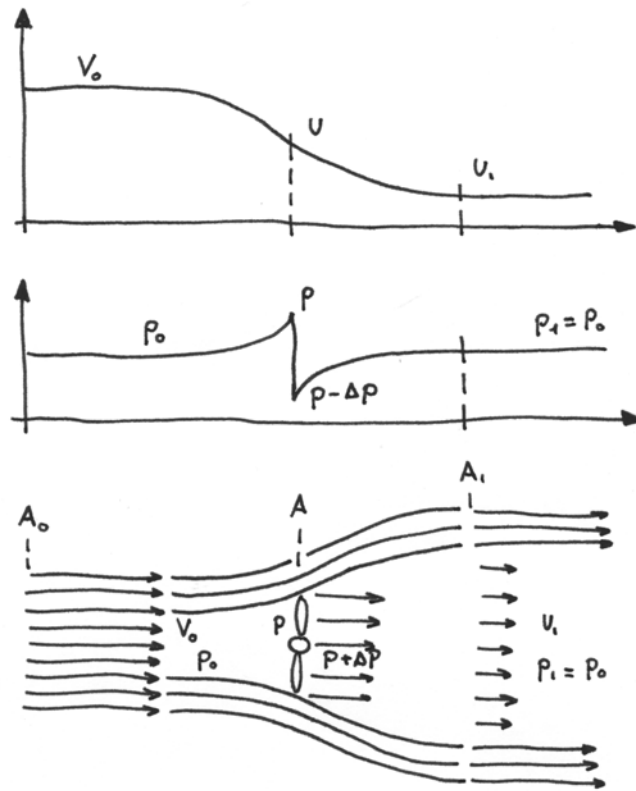


Figure 2: Flow near an idealized turbine: velocity and pressure.

$$\begin{aligned}
 p_0 + \frac{1}{2}\rho V_0^2 &= p + \frac{1}{2}\rho u^2 & \text{and} \\
 p - \Delta p + \frac{1}{2}\rho u^2 &= p_0 + \frac{1}{2}\rho u_1^2
 \end{aligned}
 \tag{2}$$

These two equations are then subtracted to yield the drop in pressure over the rotor plane

$$\Delta p = \frac{1}{2}\rho(V_0^2 - u_1^2)
 \tag{3}$$

Another method for calculating the drop in pressure,  $\Delta p$ , is expressing the drop as the change in momentum of the mass of air passing through one square meter of the rotor area per second (actually by considering the second law of Newton). This yields

$$\Delta p = \rho u(V_0 - u_1)
 \tag{4}$$

Now equating the equations (3) and (4) gives an expression for the wind speed in the rotor plane:

$$u = \frac{1}{2}(V_0 + u_1)
 \tag{5}$$

i.e. the velocity in the rotor plane is exactly the average of the far upstream and the far downstream wind speed.

## 1. Introduction to Wake Modelling

The axial thrust force – i.e. the force acting in the direction of the wind – which is denoted,  $T$ , is calculated from knowledge of the pressure difference:

$$T = \Delta p \cdot A \quad (6)$$

where  $T$  is the thrust force  
 $\Delta p$  is the difference force  
 $A$  is the rotor area

Now defining a ‘axial interference factor’ –  $a$ , which is

$$u = (1 - a)V_0 \quad \text{and thus} \quad u_1 = (1 - 2a)V_0 \quad (7)$$

Inserting the equation (7) and equation (3) into the thrust definition equation (6) yields

$$T = 2\rho a(1 - a)V_0^2 A \quad (8)$$

Now defining a thrust coefficient,  $C_T = 4a(1 - a) \Rightarrow a = 1/2 \pm 1/2\sqrt{1 - C_T}$ , gives

$$T = (1/2) \cdot \rho \cdot V_0^2 A \cdot C_T \quad (9)$$

Inserting the expression for  $a$  (and  $C_T$ ) into the equation (7), this yield

$$u_1/V_0 = (1 - 2a) = \sqrt{1 - C_T} \quad (10)$$

With the equation (10) we now have a relation established between downstream wake wind velocity –  $u_1$ , the turbine thrust coefficient –  $C_T$  – and the free wind speed –  $V_0$ . This relation is - using an assumption of the downstream wake expansion – used for making simple and computationally very efficient turbine wake models – like the N.O. Jensen PARK model.

Wake Expansion: When applying the continuity equation in relation with the equation (6) and (10) which are expressions for the wind speed in the rotor plane ( $u$ ) and far downstream of the turbine ( $u_1$ ), then an expression for the so-called expanded diameter can be derived [2]:

$$D_{\text{exp}} = RD\sqrt{(1 - a)/(1 - 2a)} \quad (11)$$

Turbulent mixing makes the wind speeds recover to the free wind speeds at some downstream distance, but the equations (10) and (11) can be used to gain insight in the wake expansion rate.

### Important Wake Model Parameters

The wake models require different internal wake model parameters as input - as well as a varying number of additional parameters describing the terrain and/or wind climate conditions. Input parameters to a wake model can be turbulence intensity and roughness length. Typically, one would assume that such parameters are depended on the roughness class (or roughness length). In the lack of the preferred measured data, the table below suggests corresponding estimated wake model parameters.

## 1. Introduction to Wake Modelling

| Terrain classification | Roughness Class | Roughness Length | Wake Decay Constant | Ambient Turbulence at 50 m* | Additional detailed description   |
|------------------------|-----------------|------------------|---------------------|-----------------------------|---|
| Offshore. Water areas  | 0.0             | 0.0002           | 0.040               | 0.06                        | Water areas, oceans and large lakes. General water bodies.                  |
| Mixed water and land   | 0.5             | 0.0024           | 0.052               | 0.07                        | Mixed water and land. Also applies to the very smooth terrain               |
| Very open farmland     | 1.0             | 0.03             | 0.063               | 0.10                        | No crossing hedges. Scattered buildings. Smooth hills.                      |
| Open farmland          | 1.5             | 0.055            | 0.075               | 0.11                        | Some buildings. Crossing hedges with 8 m height with distance 1250 m apart. |
| Mixed farmland.        | 2.0             | 0.10             | 0.083               | 0.12                        | Some buildings. Crossing hedges 8 m high with distance 800 m apart          |
| Trees and farmland     | 2.5             | 0.20             | 0.092               | 0.13                        | Closed appearance. Dense vegetation. 8 m hedges 250 m apart.                |
| Forests and villages   | 3.0             | 0.40             | 0.100               | 0.15                        | Villages, small towns and much closed farmland. Many high hedges. Forests.  |
| Large towns and cities | 3.5             | 0.80             | 0.108               | 0.17                        | Large towns, cities with extended build up areas.                           |
| Large build up cities  | 4.0             | 1.60             | 0.117               | 0.21                        | Large cities with build up areas and high buildings.                        |

\* The turbulence intensity is actually calculated based on the assumption of homogeneous terrain with a surface roughness equal to the roughness length. Input to the calculation is also the turbulence measurement height – see the equation below.

### Estimating the Turbulence Intensity

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain - be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [3, section 3.1.2]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[ \frac{1}{\ln[z / z_0]} \right]$$

The value of A is reported to vary approximately between 2.5 to 1.8.  $\kappa$  is the Von Karman constant, which is equal to 0.4. In DS 472 the product between  $A_x$  and  $\kappa$  is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC, the characteristic data needed is actually a mean value plus one standard deviation.

### Wake Model Limitations – Large Wind Farms and Thrust Coefficient

The wake models are calibrated and tested in small to medium sized wind farms – i.e. wind farms with up to approximately 50-75 turbines. For very large wind farms – 75 turbines or more – the turbines may influence the surrounding upper air wind climate (Geostrophic winds). In that case, special modelling should be applied – e.g. by ‘artificially’ increasing the roughness within the wind farm.

An important parameter for most models is the thrust coefficient,  $C_t$ . The thrust coefficient is used to relate the free wind speed to the downstream wake wind speed through the equation  $u_1/V_0=(1-C_t)^{0.5}$ . Since the square root is taken, it might be a requirement – depended on the wake model - that the value of  $C_t$  is less than 1.0.

## **References**

- [1] P.S. Andersen, U. Krabbe, P. Lundsager, H. Petersen, *Basismateriale for Beregning af Propelvindmøller*, Risø-M-2153(rev.), Forsøgsanlæg Risø, Januar 1980.
- [2] J.G. Schepers: *ENDOW: Validation and improvement of ECN's wakemodel*, ECN-C-03-037, March 2003.
- [3] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition, Risø National Laboratory

## 2. Introduction to the N.O. Jensen Wake Model

The N.O. Jensen wake model is a simple single wake model. The model is documented in the paper ‘A Simple Model for Cluster Efficiency’ by I. Katić et al [1] and is based on the assumption of a linearly expanding wake diameter. This note gives an introduction to the N.O. Jensen wake model and how it is implemented in WindPRO.

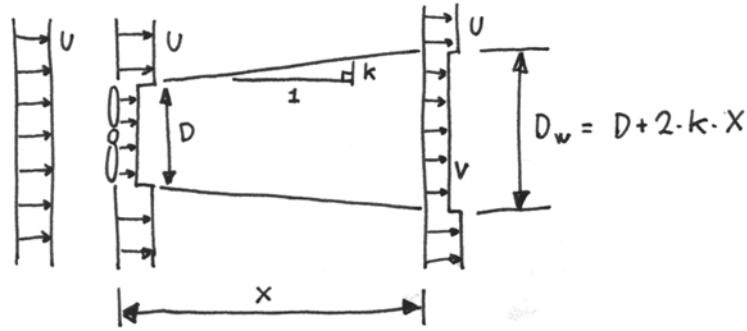


Figure 1: The N.O. Jensen wake model - overview.

### Single Wake Calculation

When calculating the velocity deficit, the reduced wind speed,  $V$ , downwind of the turbine is derived from:

$$1 - V/U = (1 - \sqrt{1 - C_T}) / (1 + 2kX/D)^2 \quad (1)$$

Comparing this equation with the previously derived equation for the wind speed just downwind of the turbine (2), it is obvious that the assumption is a linearly expanding wake width.

$$V_0/U = \sqrt{1 - C_T} \quad (2)$$

where

$V_0$  is the wind speed directly after the turbine of consideration

However it is noticed, that it is not the actual wake wind velocity that is subject for this expansion assumption, but rather the velocity deficit  $\delta V_i = (1 - V_i/U)$ . Note, that the velocity deficit is defined through the free wind speed,  $U$ .

A plot from a calculation in WindPRO is shown in Figure 2. Note, that the wake velocity deficit is uniform given a certain downstream position.

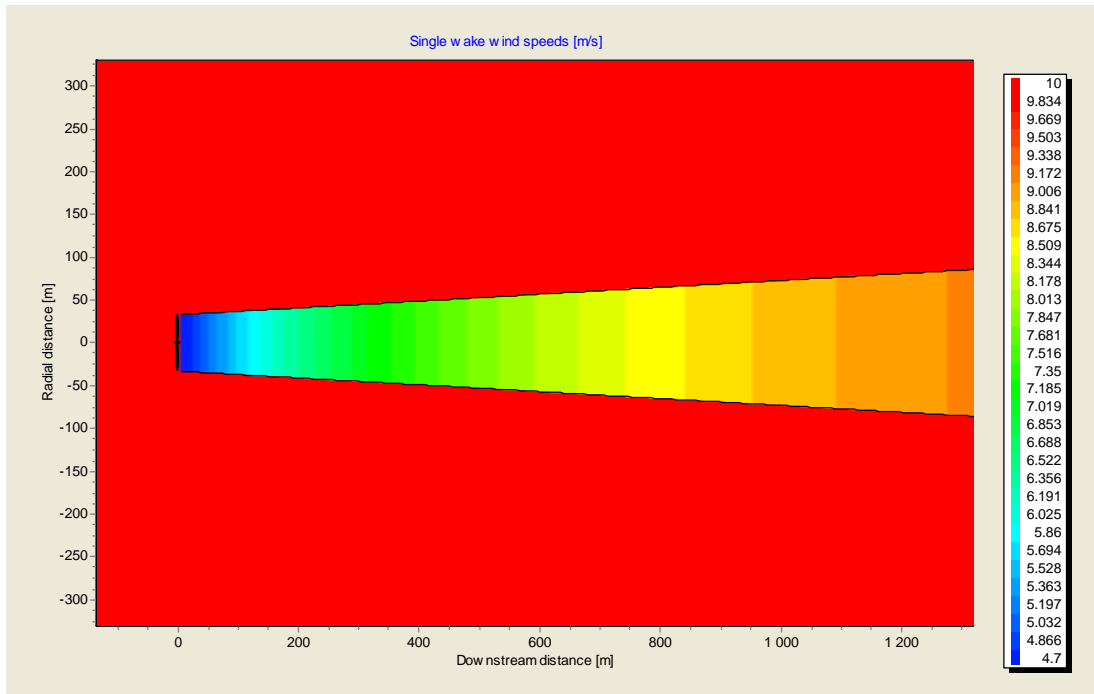


Figure 2: N.O. Jensen wake model - wake development after a single turbine.

## Multiple Wake Calculation

Katic et al. [1] suggests, that multiple wakes are calculated through the ‘sum of squares of velocity deficits’ wake combination model. Thus, the N.O. Jensen model initially implemented in the WindPRO PARK module as well as the WAsP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution. The combined effects of multiple wakes are found as:

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2}$$

This model is treated in a succeeding chapter.

## References

- [1] I. Katić, J. Højstrup & N.O. Jensen, *A Simple Model for Cluster Efficiency*, European Wind Energy Association, Conference and Exhibition, 7-9 October 1986, Rome, Italy.

### 3. Introduction to the Ainslie Wake Model (Eddy Viscosity Model)

#### Introduction

The wind turbine wake application of an axi-symmetric formulation of the time averaged Navier Stokes equations with an eddy viscosity closure was initially made by Ainsley [3]. The application uses cylindrical coordinates and an assumption of incompressible fluid. A graphical overview of the model setup is shown in Figure 1.

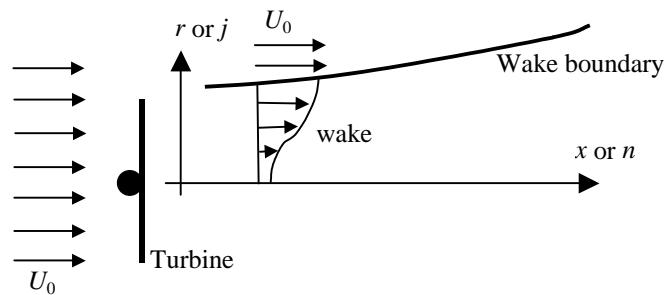


Figure 1: Flow around the turbine.

A result from an implementation of the model is shown in Figure 2, where the wake development behind a turbine with a 50 meters rotor is shown. The calculation is initiated at distance two-rotor diameters downstream (100 meters). Note, that the model calculates the flow through half of the rotor as indicated on Figure 1), this is due to the symmetry assumption used within the model. The free stream velocity is 8.0 m/s (as shown in the legend to the left), while the minimum velocity behind the turbine is 6.5 m/s. The trust coefficient – for this sample calculation – was set to 0.7.

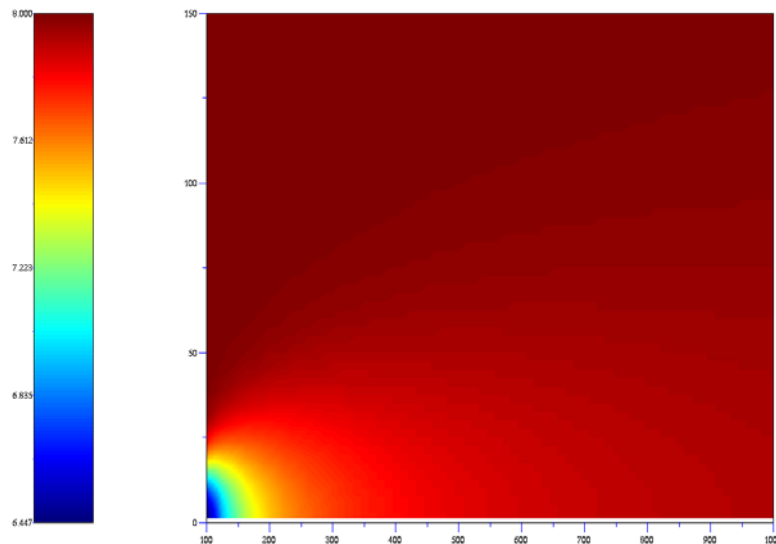


Figure 2: Wake development behind a turbine with a 50-meter rotor. Note that the calculation is initiated at 2·RD downstream.

#### Nomenclature

|       |  |
|-------|--|
| $I_T$ | turbulence intensity                         |
| $v$   | mean (averaged) velocity in radial direction |
| $u$   | mean (averaged) velocity in axial direction  |
| $U_0$ | mean (averaged) velocity in free flow        |

|                  |  |
|------------------|--|
| $\sigma_u$       | standard deviation of wind speed process |
| $uv$             | Reynolds stress                          |
| $\varepsilon(x)$ | eddy viscosity                           |

### Continuity Equation (axisymmetric case: $\partial/\partial\phi = 0$ )

The continuity equation in cylindrical coordinates is (Shames [1]):

$$\frac{1}{r} \frac{\partial rv}{\partial r} + \frac{\partial u}{\partial x} = 0$$

### Navier Stokes Equation

In the thin layer approximation and using cylindrical coordinates, the Navier Stokes equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{1}{r} \frac{\partial(\overline{ruv})}{\partial r}$$

The last part of the equation above refers to the change in acceleration and thereby momentum. It is not possible to describe this contribution using the velocities in the averaged flow. The part is due to the change in momentum caused by the turbulent fluctuations.

### Eddy viscosity (or turbulent exchange coefficient for momentum)

The eddy viscosity is defined in Tennekes and Lumley [2], and is used for establishing an interaction between mean flow and turbulent eddies.

$$-\overline{uv} = \varepsilon(x) \frac{\partial u}{\partial r}$$

According to Ainslie [3], the eddy viscosity,  $\varepsilon(x)$ , is adequately described by a length scale  $L(x)$  and a velocity scale  $U(x)$ .

$$\varepsilon(x) = L(x) \cdot U(x) + \varepsilon_a$$

The length and velocity scales are taken to be proportional to the wake width  $b$  and the velocity difference across the wake shear layer (i.e. independent of  $r$ ).  $\varepsilon_a$  is the contribution from ambient turbulence to the eddy viscosity. The length scales are determined by:

$$L(x) \cdot U(x) = k_1 \cdot b \cdot (U_0 - u_0(x))$$

3 case studies showed  $k_1 = 0.015$ .

### Boundary conditions

Ainsley [3] gives the boundary conditions at two rotor diameters downstream of the turbine. The BC at this section is given as a Gaussian velocity profile with the input of initial velocity deficit  $D_M$  and wake width  $b$ :

$$1 - \frac{U}{U_0} = D_M \exp\left[-3.56 \cdot \left(\frac{r}{b}\right)^2\right]$$

Empirical data (wind tunnel studies) showed the following equations may be used for determining the velocity deficit and the wake width ( $A$  is ambient turbulence intensity in percent):

$$D_M = C_T - 0.05 - (16C_T - 0.5)A / 1000$$

$$b = \frac{3.56C_T}{8D_M(1 - 0.5)D_M}$$

Other authors specify a boundary condition where the initiation position (downwind position) varies. In Lange et al [4] reference to a study made by Vermeulen [5] is made. Vermeulen suggests that the near wake length is modeled through contributions from ambient turbulence, rotor generated turbulence and shear generated turbulence. The near wake length is divided into two regions; the first  $x_h$  is modeled as:

$$x_h = r_0 \left[ \left( \frac{dr}{dx} \right)_a^2 + \left( \frac{dr}{dx} \right)_\lambda^2 + \left( \frac{dr}{dx} \right)_m^2 \right]^{-0.5}$$

where  $r_0$  is an 'effective' radius of an expanded rotor disc,  $r_0 = [D/2] \sqrt{(m+1)/2}$  and  $m = 1/\sqrt{1-C_t}$   
 $D$  is the rotor diameter  
 $C_t$  the thrust coefficient

The different contributions in the equation above are calculated as:

$$\left( \frac{dr}{dx} \right)_a^2 = \begin{cases} 2.5I + 0.05 & \text{for } I \geq 0.02 \\ 5I & \text{for } I < 0.02 \end{cases} \quad \text{ambient turbulence}$$

$$\left( \frac{dr}{dx} \right)_\lambda^2 = 0.012B\lambda \quad \text{rotor generated turbulence}$$

$$\left( \frac{dr}{dx} \right)_m^2 = [(1-m)\sqrt{1.49+m}] / (9.76(1+m)) \quad \text{shear - generated turbulence}$$

where  $I$  is the ambient turbulence intensity  
 $B$  is the number of rotor blades  
 $\lambda$  is the tip speed ratio

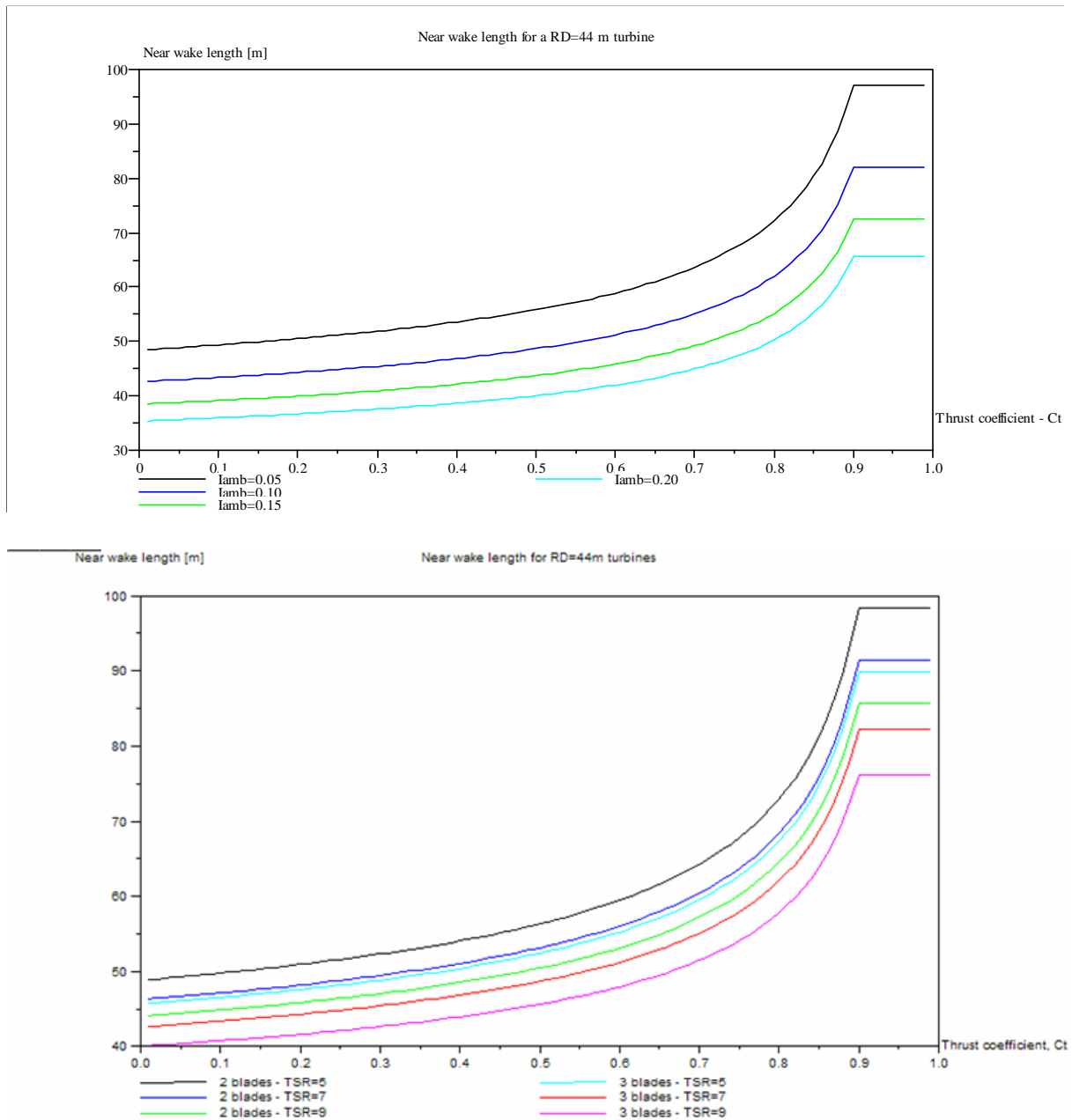


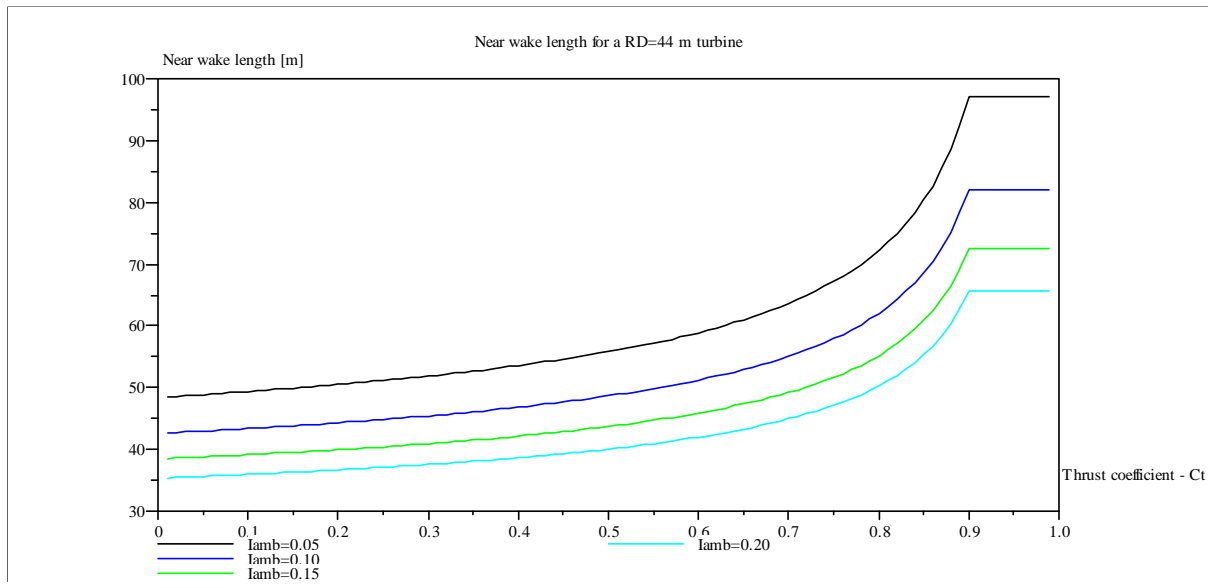
Figure 3: Near Wake Length for a 44 meter Rotor-Diameter Turbine using the Vermeulen Equations. Top: Sensitivity to ambient turbulence. Bottom: Sensitivity to type of turbine and tip speed ratio.

When the first near wake region,  $x_h$ , have been calculated, one can calculate the full near wake length,  $x_n$ , by:

$$x_n = \frac{\sqrt{0.212 + 0.145m}}{1 - \sqrt{0.212 + 0.145m}} \frac{1 - \sqrt{0.134 + 0.124m}}{\sqrt{0.134 + 0.124m}} x_h$$

Lange [4] reports that the equations save a singularity at about  $C_T=0.97$ , so it is suggested that for  $C_T$ 's larger than 0.9, then the value for  $C_T$  equal to 0.9 is used. A sample calculation for a 44-meter rotor diameter turbine is shown in

Introduction to the Ainslie Wake Model (Eddy Viscosity Model)



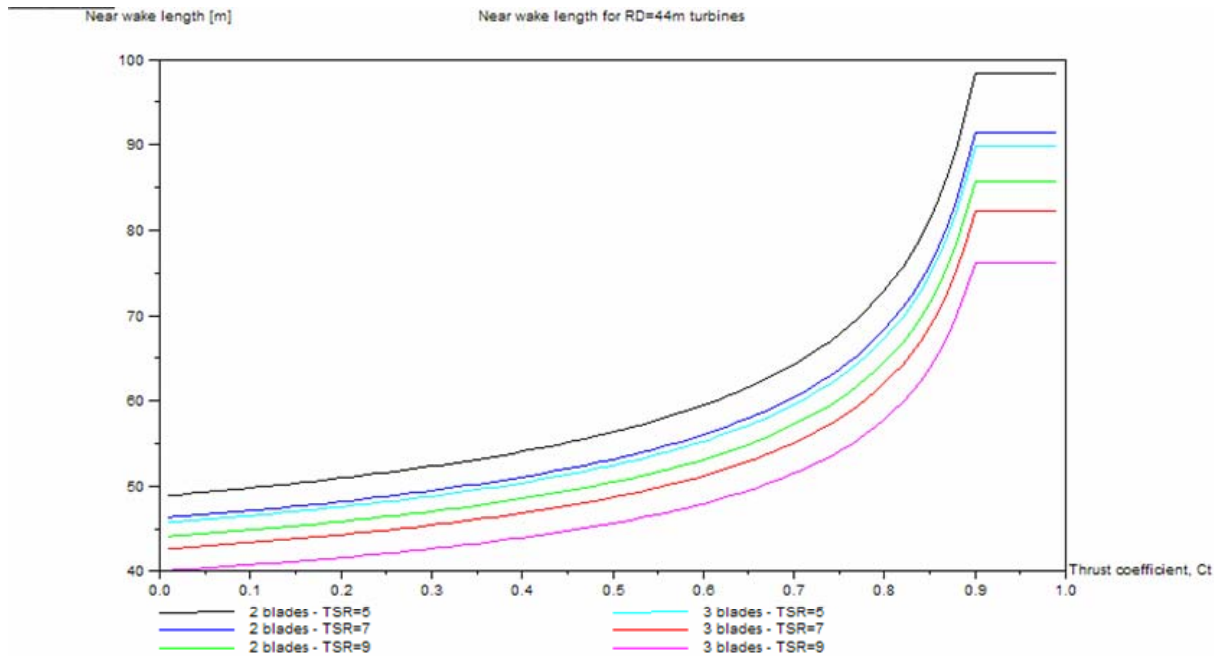


Figure 3. Note, that the near wake length is decreasing with increasing ambient turbulence levels.

### Numerical solution method

The differential equation is solved using a finite difference method using a generalized Crank-Nicholson scheme. The solution procedure followed is outlined in Wendt [6]. The numerical solution method used for solving the Navier Stokes equation is made by replaced the differential equation with the finite difference approximations. This approximation introduces truncation errors into the equation.

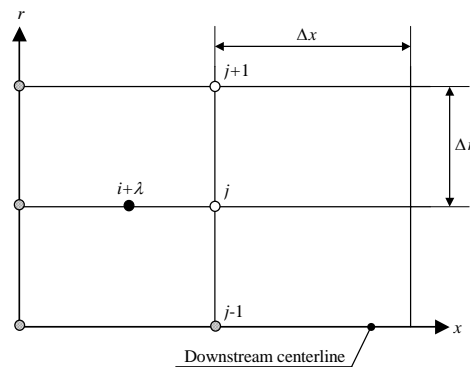


Figure 4: Grid for the generalized implicit method.

### Outline of the Solution Procedure

The solution of the partial differential equations invokes an iterative solution procedure. From the boundary condition, the continuity equation is solved. Then the downstream momentum equation is solved in order to get the next downstream velocities. This solution is obtained through an iterative process – the iteration is stopped when convergences is achieved.

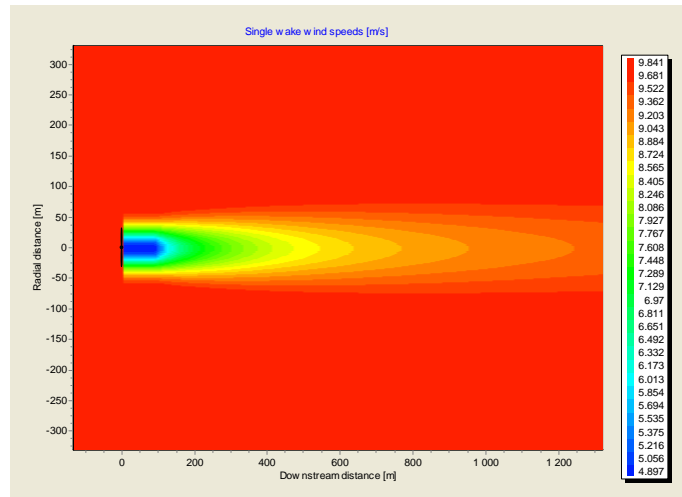


Figure 5: Eddy viscosity model – calculation from WindPRO.

A plot from WindPRO is shown in Figure 5. Note, that wind speeds within the near wake zone are approximated through the solution at the near wake distance.

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## 4. Introduction to the G.C. Larsen Model (EWTS II)

### Introduction

This model is a semi analytical model – derived from asymptotic expressions from Prandtl’s rotational symmetric turbulent boundary layer equations. Because of the asymptotic expressions, the model might be somewhat conservative for close spacings. The model is reported in [1] and is also the recommended wake model – for use with wake loading - in the project report from the European Wind Turbine Standards II Project, an EU-funded project finalized in 1999 [2]. This introduction is based on the EWTS-report [2]. A online introduction of the model can be found in the Risø report ‘A simple wake model’ [3] – this report is available online.

### Model Equations

Assuming that similarity exist between deficits at different downstream positions and only moderate velocity deficits, then the wake radius can be described by:

$$R_w = \left[ \frac{35}{2\pi} \right]^{1/5} [3c_1^2]^{1/5} [C_T Ax]^{1/3} \quad (1)$$

where

$c_1$  is a non-dimensional mixing length, described by  $c_1 = l(C_T Ax)^{-1/3}$   
 $l$  is Prandtl’s mixing length

The  $c_1$  parameter does – according to reference [2] – to some degree separate the rotor drag dependence and thus the  $c_1$  is expected to be relative insensitive to the design and size of the rotor. An alternative and approximated specification of the  $c_1$  parameter is found in section 5.1 of the EWTS II report [2], where the parameter is estimated as seen below in equation (2). This specification is adopted in WindPRO.

$$c_1 = \left[ \frac{D}{2} \right]^{-1/2} (C_T Ax_0)^{-5/6} \quad (2)$$

where

$C_T$  is the thrust coefficient  
 $A$  is the rotor area  
 $D$  is the diameter of the upstream rotor  
 $x_0$  is an approximation parameter, determined by the equation (3) below

$$x_0 = 9.5D / \left( \frac{2R_{9.5}}{D} \right)^3 - 1 \quad (3)$$

In the equation above the  $R_{9.5}$  parameter is determined as:

$$\begin{aligned} R_{9.5} &= 0.5[R_{nb} + \min(h, R_{nb})] \\ R_{nb} &= \max(1.08D, 1.08D + 21.7D(I_a - 0.05)) \end{aligned} \quad (4)$$

where

$I_a$  is the ambient turbulence intensity at hub height

The wake boundary condition is satisfied, so that the wake radius at the rotor position equals the rotor diameter. Furthermore, empirical boundary condition is applied at 9.5 rotor diameters downstream, where the wake radius is determined from the equation (4). The equation ensures that the minimum turbulence intensity equal to 5% is used, and it essentially states that the wake expansion is dominated by ambient turbulence. The blocking effect of the ground is taken into account by using the design wake radius  $R_{9.5}$  in eq. (4) – including the mean of  $R_{nb}$  and the minimum of the hub height and  $R_{nb}$ .

Mean Wind Velocity Deficit: The mean wind deficit is determined from the expression (5),

$$\Delta V = -\frac{V_a}{9} (C_T Ax^{-2})^{1/3} \left\{ r^{3/2} (3c_1^2 C_T Ax)^{-1/2} - \left( \frac{35}{2\pi} \right)^{3/10} (3c_1^2)^{-1/5} \right\}^2 \quad (5)$$

where

$V_a$  is the ambient mean wind velocity at hub height

### Modified Near Wake Description

The G.C.Larsen wake model includes the option of having a semi-empirical near wake description (second order approach) – enabling the user to model the near wake with a ‘double peak’ velocity profile. This approach is described in detailed in reference [1]. Using the second order option may give a more precise near wake description, especially for densely space turbines. The far wake is not modified.

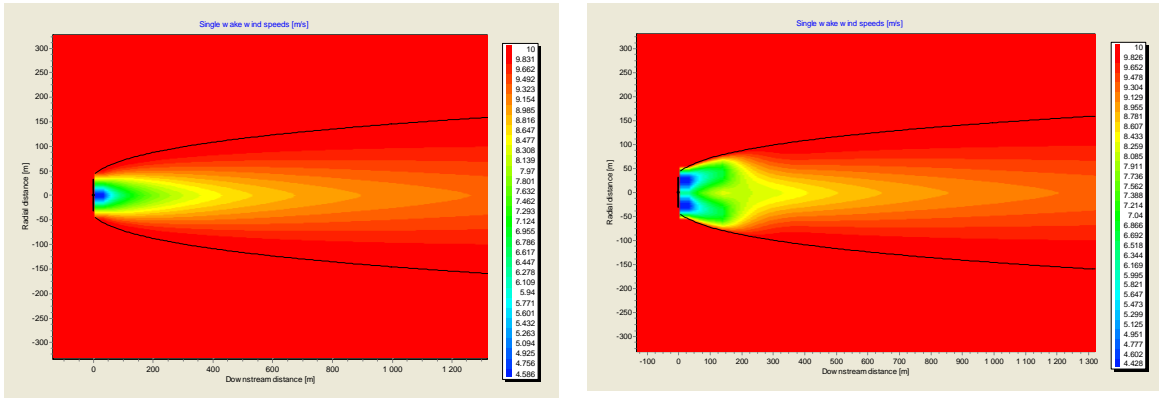


Figure 1: G.C.Larsen wake model (left: First order approach, right: second order approach)

### References

- [1] G.C.Larsen, J. Højstrup, H.A. Madsen, *Wind Fields in Wakes*, EUWEC '96, Gothenburg, 1996.
- [2] *European Wind Turbine Standards II*, ECN-C-99-073, 1999
- [3] Larsen, G.C, *A simple wake calculation procedure*, RISØ-M—2760, Risø National Lab., Roskilde (Denmark), (online: <http://www.risoe.dk/rispubl/VEA/veapdf/ris-m-2760.pdf>)

## 5. Wake Combination Models

Today (2005), most wake models are still single wake models. Thus, in order to obtain a usable result for wind farms with many turbines, these single wakes must be combined into a combined effect. This is done by purely empirical means, using different wake combination models.

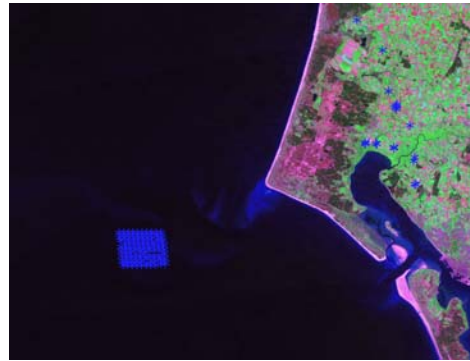


Figure 1: Horns Rev offshore wind farm.

### Introduction

Two problems occur when trying to combine the results from a several single wake model into one single downwind wind speed:

1. Since the results from many of the single wake models are non-uniform distributed velocities or velocity deficits, these results must be averaged or combined into an efficient (uniform) wind speed. This is necessary, because the wind turbine power output is to be estimated from the available power curves.
2. When the downwind velocities are determined through one single wake calculation for each turbine, the single wake results must be added into a combined effect.

### Ad. 1: Averaging of the Single Wake results

The output from many wake calculations is a non-uniform velocity field. However in order to calculate the power output from a measured power curve, the velocity field must be averaged over the rotor area. In WindPRO, a squared momentum deficit approach is used to calculate this reduction. This approach is similar to the one reported by Lange et. al [1].

$$(u_0 - u_{rotor})^2 = \frac{1}{A} \int_{rotor} (u_0 - u_w)^2 dA \quad (1)$$

where  $u_0$  is the free stream velocity  
 $u_{rotor}$  is the averaged velocity at the rotor  
 $u_w$  is the non-uniform wake velocity (i.e. a function of the distance and direction from the hub)

Investigations made in connection to the validation of the wake models implemented showed, that using linear combination of wind speeds or using exponents of order 3 only gave marginal differences on the averaged wind speed. The integration in (1) is done by numerical means.

### Ad. 2: Wake Combination Models

This averaging may be done in a variety of combinations. Djerf [2] states on option of four different wake combination methods: 1) Sum of squares of velocity deficits, 2) Energy balance, 3) Geometric sum, 4) Linear superposition. According to Djerf it is not recommend using methods (3) and (4). Schepers [3] suggests another approach. Schepers first calculates the wake from the upstream turbine. Then this wake is used for calculating the axial force coefficient on the second turbine downstream. The initial velocity deficit behind the second turbine is then calculated from the axial force.

In WindPRO, the ‘Sum of squares of velocity deficit’ methodology is used.

## Sum of Squares of Velocity Deficits

The N.O. Jensen model initially implemented in the WindPRO Park module as well as the WASP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution.

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2} \quad (2)$$

where  $\delta V$  is the velocity deficit defined as  $(1-V/U)$  – where  $U$  is the free wind speed  
 $n$  is the number of upstream turbines

Lange et. al [1] uses a slightly different formulation of the sum of the squares of velocity deficits approach. This equation is used in conjunction with equation (1) to calculate the deficit.

$$(u_0 - u_c)^2 = u_0^2 \left(1 - \frac{u_c}{u_0}\right)^2 = \frac{1}{A} \int_{rotor\ i, allwakes} \sum (u_{rotor(i)} - u_{w(i)})^2 dA \quad (3)$$

## Outline of the calculation procedure

An overview of the calculation procedure is as follows:

1. The calculation is initiated with the turbine positioned at the most upstream position (lup turbine)
2. Find (calculate) the wind speed directly upstream of the turbine
3. Calculate the wind speeds downstream of this turbine, i.e. for all downwind turbine positions
4. Calculate the deficits for all downstream turbine positions, i.e. relating to the free wind speed
5. If the downstream turbine is in a partial wake, then reduce the velocity deficit with the fraction of the overlap area to the rotor area of the downstream turbine.
6. Calculate the square of the velocity deficits
7. Continue with the next turbine (using step 1), by summing the squares of the velocity deficits.

## References

- [1] Lange, Bernard et al: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLAP*, Wind Energy 2003; 6: 87-104  
 [2] Djerf, E. & Mattson, H.: *Evaluation of the Software program WindFarm and Comparisons with Measured Data from Alsvik*, FFA TN-2000-30,  
[http://www.vindenergi.foi.se/Rapporter/TN2000\\_30\\_WindFarm.pdf](http://www.vindenergi.foi.se/Rapporter/TN2000_30_WindFarm.pdf)  
 [3] Schepers, *ENDOW: Validation and Improvement of ECN's wake model*, ECN-C-03-034.

## 6. Introduction to Turbulence and Wakes

Turbines operating in wakes are subjected to higher turbulence loading than turbines operating in the free wind. Appropriate turbulence calculations should be made before selecting the proper turbine design class when having clusters of turbines. The wake added turbulence may be calculated using different wake or turbulence models. These models are typically very different in detailing level – and possible also in accuracy. The models range is from simple engineering models to the more advanced computational fluid dynamic (CFD) models. The CFD-models are typically also very demanding in terms of calculation time.

This chapter gives an introduction to the operations performed on the measured turbulence data, and how the turbulence data from single wake models is merged.

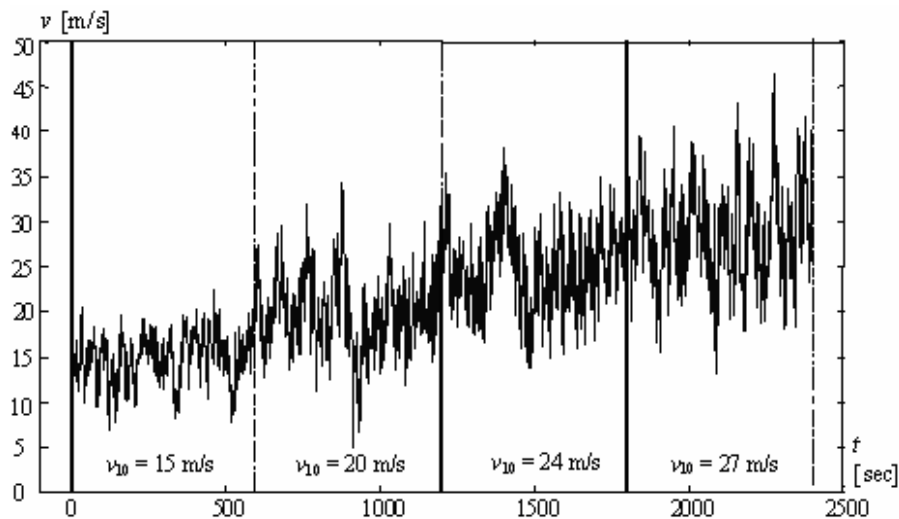


Figure 1: Turbulent winds – simulated at different mean wind speed, from [7].

WindPRO offers calculation on the wake added turbulence. The ambient turbulence level must be user specified, e.g. through measured data or alternatively through the roughness classification.

### The Turbulence Calculation

When calculating the design, lifetime and fatigue on wind turbines, the turbulence levels are of utmost importance. The turbulent winds arise from several sources:

1. Orography induced turbulence, i.e. flow over hills and mountains
2. Roughness induced turbulence, i.e. flow generated by objects within the landscape
3. Turbine generated turbulence, i.e. turbulence in the wake of the turbines

The turbulence intensity is defined as the ratio between the standard deviation of the wind speed,  $\sigma_u$ , and the 10-minute mean wind speed,  $U_{10}$ . When dealing with wind turbine wakes, it is tradition to relate the 10-minute mean wind speed to the free wind speed, i.e. the wind speed outside the wake.

$$I_T = \frac{\sigma_u}{U_{10}}$$

The current edition of WindPRO – version 2.5 – deals primarily with the turbine generated turbulence. Orography and roughness generated turbulences are included only through the on-site meteorological measurements – or alternatively through user-defined turbulence input levels.

## Vertical Scaling of the Ambient Turbulence Level

Often, when turbulence measurements are available from the site, the measurements are not taken at hub-height level. This calls for a vertical scaling of the ambient turbulence, which is done by assuming homogeneous terrain (an approximation to the real nature). Preferably, the turbulence should be taken from hub-height measurements.

Assuming that the wind flow is a horizontally homogeneous (i.e. the properties of the flow do not change in the horizontal direction), then the standard deviation of the wind speed process is only depended of the height above the terrain,  $z$ .

The turbulence intensity in the height  $x$  meters is defined as:

$$IT(x) = \frac{\sigma_U(x)}{U_{10}(x)} \quad (1)$$

Where

$IT$  is the turbulence intensity

$\sigma_U$  is the standard deviation of the wind speed

$U_{10}$  is the mean wind speed averaged over 10 minutes

Experimental data has shown that the standard deviation of the wind speed only decreases very slowly. In Armit [1] & Dyrbye & Hansen [2], it is said, that it is reasonable to use constant standard deviations up to about the half-height of the internal boundary layer. This assumption is also used in WAsP and in most structural codes.

Using this assumption, the vertical scaling of turbulence intensity between two heights is simply calculated by assuming the same standard deviations in the two heights ( $x$  and  $y$  meters or feet).

$$\begin{aligned} \sigma_U(x) &= \sigma_U(y) && \Leftrightarrow \\ IT(x) \cdot U_{10}(x) &= IT(y) \cdot U_{10}(y) && \Leftrightarrow \\ IT(y) &= \frac{U_{10}(x)}{U_{10}(y)} IT(x) \end{aligned} \quad (2)$$

So now the problem is reduced into calculating the mean wind speed in the new height. The vertical scaling of wind speeds may be done using the power law vertical wind profile a purely empirical equation. The power law wind profiles also require quite homogenous terrain.

$$U_{10}(y) = U_{10}(x) \cdot \left[ \frac{y}{x} \right]^\gamma \quad (3)$$

where

$\gamma$  is the wind gradient exponent

The wind gradient exponent is known to be very depended on the roughness length or the roughness class. The table below gives guidelines for selecting the wind gradient exponent – if no measured data is available:

| Roughness Class | Roughness Length | Wind Gradient Exponent |
|-----------------|------------------|------------------------|
| 0               | 0.0002           | 0.1                    |
| 1               | 0.03             | 0.15                   |
| 2               | 0.1              | 0.2                    |
| 3               | 0.4              | 0.3                    |

Inserting the equation (3) into (2) we obtain the turbulence scaling law, valid for homogeneous terrain:

$$IT(y) = \frac{U_{10}(x)}{U_{10}(y)} IT(x) = U_{10}(x) \cdot IT(x) \cdot \left[ \frac{y}{x} \right]^{-\gamma} \quad (4)$$

### Calculating turbulence intensity from roughness data

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain - be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [4]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[ \frac{1}{\ln[z / z_0]} \right] \quad (5)$$

The value of A is reported to vary approximately between 2.5 to 1.8.  $\kappa$  is the Von Karman constant, which is equal to 0.4. In DS 472 [5] the product between  $A_x$  and  $\kappa$  is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC 61400-2, the characteristic data needed is actually a mean value plus one standard deviation.

### Turbulence from Wind Turbine Wakes

The wake added turbulence is either derived from the (single) wake models that include turbulence modelling or from dedicated (empirical) turbulence models. The turbulence calculated from the different models may be parameterized in numerous ways, see e.g. Figure 2 which holds output from the eddy Viscosity wake model. Using the EV-model one may relate the eddy viscosity to the turbulence intensity or alternative use empirical values. Again, other models have wake turbulence included in a purely empirical manner. The turbulence model must be used in connection with a wake model – in order to take the reduced wind speeds in the wind farm into account.

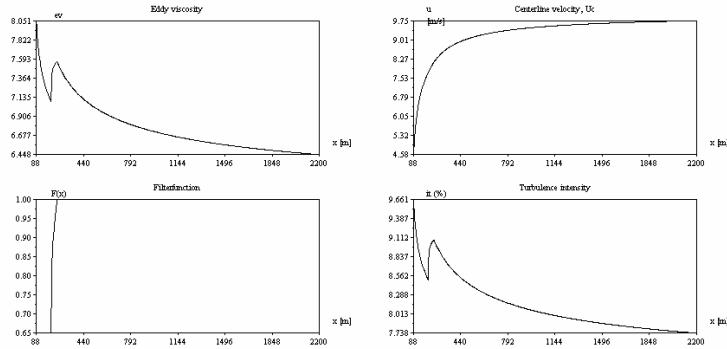


Figure 2: Single Wake Turbulence Modelling using the Eddy Viscosity Model.

The results from the turbulence models – may typically come within one of four categories:

1. Added turbulence model – calculated for the wake after a single turbulence
2. Added turbulence model – calculated for all surrounding turbines
3. Total turbulence model - calculated for the wake after a single turbulence
4. Total turbulence model – calculated for all surrounding turbines

Models (1) and (2) give the wake added turbulence contribution. This should be added to the ambient turbulence level. The model type (3) gives the total turbulence level for a given wake at a given position (ambient and wake added), and this must be summed into a combined effect considering all upstream turbines. The model type (4) gives the total turbulence level in an integrated manner, thus no single wake adding is needed.

All of the turbulence models implemented in WindPRO belong to any of these four types.

### Calculating the wake added turbulence intensity

The turbulence intensity is defined as the ratio of standard deviation to the mean wind speed. It is common practice to relate the turbulence intensity – also within the wake – to the ambient free wind speed. Also, it is practice to assume that the added turbulence level may be added as independent stochastic variables.

In the Danish Recommendation [6] the total turbulence intensity is actually calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (6)$$

### Partial Wakes - Turbulence

When the turbine operates in a partial wake, we use the equation (6) to calculate the added turbulence level – considering the rotor area with ambient turbulence only. A linear weighting with rotor areas is assumed.

### References

- [1] Armit, *Wind Structures*, Lecture Series, Von Karman Institute for Fluid Dynamics, 1976.
- [2] Dyrbye & Hansen, *Wind Loads on Structures*, John Wiley and Sons, 1996

## 7. Danish Recommendation – Turbulence Model

The Danish Recommendation [1] from 1992 specifies a quite simple wake added turbulence model. If the turbines are erected in a cluster with a minimum distance between the turbines of 5 times the rotor diameter – or in a row with the distance 3 times the rotor diameter – then a added turbulence intensity of  $I_{park} = 0.15$  can be used. An alternative is to use the a mean-contribution, which varies by the mean wind speed and the distance between the turbines:

$$I_{park} = \beta_v \cdot \beta_l \cdot 0.15 \quad (1)$$

where  $\beta_v$  is a parameter taking the mean wind speed into account (see the Figure 1)  
 $\beta_l$  is a parameter taking the distance between the turbines into account (Figure 2 and 3).

The  $\beta_l$  parameters are dependend on the geometrical configuration of the wind farm, i.e. if the wind farm is erected in a cluster (Figure 2 ) or in a row (Figure 3).

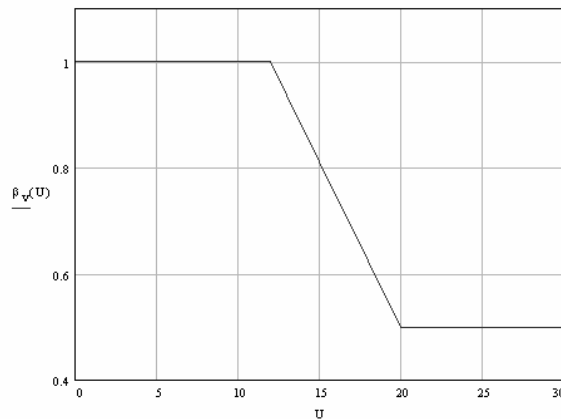


Figure 1: Factor taking wind velocity into account,  $\beta_v$ .

The  $\beta_l$  factor is determined from the Figures 2 and 3.

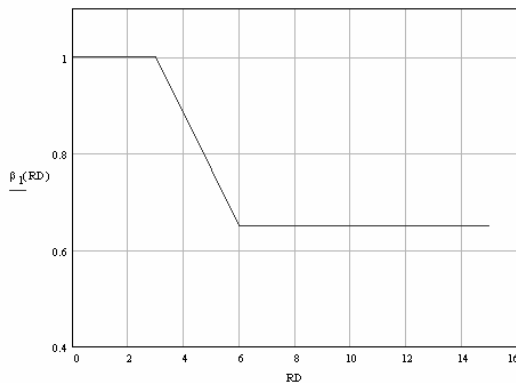


Figure 2:  $\beta_l$  for turbines in a row.

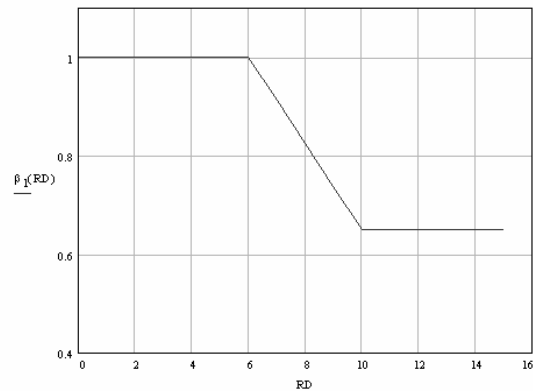


Figure 3:  $\beta_l$  for turbines in a cluster.

## **The Wake Added Turbulence**

The total turbulence intensity is calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (2)$$

## **References**

[1] *Recommendation for the fulfillment of the requirements found in the technical criteria*, Danish Energy Agency, 1992

## 8. Turbulence Model – Frandsen & DIBt

S. Frandsen and M.L. Thøgersen [1] report an empirical turbulence model for calculating the integrated wake effect of turbines. This model takes into account the different structural fatigue responses of the structural materials considered, e.g. steel in the towers and hub extenders and glass fibre reinforced polyester (GRP) or glass fibre reinforced epoxy (GRE) in the blades. The equations below assume that the wind direction is approximately uniform distributed. Reference is made to Frandsen & Thøgersen [1] and Guidelines for the Design of Wind Turbines [2].

This model is included as a recommended model in the German DIBt Richtlinie [3].

### Determining the Total Turbulence Intensity

The total turbulence intensity is determined from:

$$I_{T,total} = \left[ (1 - N \cdot p_w) I_T^m + p_w \sum_{i=1}^N I_{T,w}^m \cdot s_i \right]^{1/m} \quad (1)$$

$$I_{T,w} = \sqrt{\frac{1}{[1.5 + 0.3 \cdot s_i \cdot \sqrt{v}]^2} + I_T^2} \quad (2)$$

where  $p_w = 0.06$  (probability of wake condition)

$s_i = x_i / RD$

$N$  is the number of closest neighboring wind turbines

$m$  is the Wöhler curve exponent of the considered material

$v$  is the free flow mean wind speed at hub height

$x_i$  is the distance to the  $i$ -th turbine

$RD$  is the rotor diameter

$I_T$  is the ambient turbulence intensity (free flow)

$I_{T,w}$  is the maximum turbulence intensity at hub height in the center of the wake

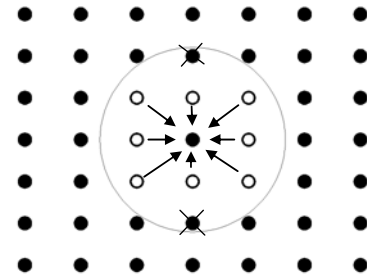
The number of closest neighboring turbines is determined as follows  
– see also the figure to the right.

$N=1$  : 2 wind turbines

$N=2$  : 1 row

$N=5$  : 2 rows

$N=8$  : Wind farms with more than two rows



### Increased Turbulence in Very Large Wind Farms

If the wind farm has more than five rows, the wind farm itself heavily influences the ambient wind climate. Also – if the distance between turbines in rows perpendicular to the predominant wind direction is less than 3 times the rotor diameter, an increase in mean turbulence level must be taken into account. This is done by substituting the ambient turbulence levels in (1) and (2) with the turbulence calculated from the equations (3) and (4).

$$I_T^* = 0.50 \cdot \sqrt{I_w^2 + I_T^2} + I_T \quad (3)$$

$$I_w = \frac{0.36}{1 + 0.08\sqrt{s_r s_f} v} \quad (4)$$

where  $s_r = x_r / RD$   
 $s_f = x_f / RD$   
 $s_r$  is the distance *within* the row  
 $s_f$  is the distance *between* rows

## References

- [1] S. Frandsen & M.L. Thøgersen, *Integrated Fatigue Loading for Wind Turbines in Wind Farms by Combining Ambient Turbulence and Wakes*, Wind Engineering, Volume 23, No. 6, 1999.
- [2] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition.
- [3] Deutsches Institut für Bautechnik – DIBt, *Richtlinie für Windenergieanlagen, Einwirkungen und Standsicherheitsnachweise für Turm und Gründung*, Fassung März 2004.

## 9. Turbulence Model – D.C. Quarton & TNO Laboratory

A simple equation to determine the wake added turbulence has been proposed by D.C. Quarton and J.F. Ainslie [1]. The parameters in the equation have been re-calibrated by Quarton and Ainslie (the modified values) and also the Dutch TNO laboratory [2].

The main form of the equation is

$$I_{add} = K_1 \cdot C_T^{\alpha_1} \cdot I_{amb}^{\alpha_2} \cdot (X / X_n)^{\alpha_3} \quad (1)$$

where  $K_1$  is a proportionality constant

$\alpha_1, \alpha_2, \alpha_3$  are exponents

$X$  is the downstream distance (in meters)

$X_n$  is the length of the near wake (see the chapter on the eddy viscosity wake model)

$I_{amb}$  is the ambient turbulence

The proportionally constant and exponents are determined from the table below

| Reference                      | $K_1$ -Constant | $\alpha_1$ -exponent | $\alpha_2$ -exponent | $\alpha_3$ -exponent |
|--------------------------------|-----------------|----------------------|----------------------|----------------------|
| Quarton and Ainslie (original) | 4.800           | 0.700                | 0.680                | -0.570               |
| Quarton and Ainslie (modified) | 5.700           | 0.700                | 0.680                | -0.960               |
| Dutch TNO laboratory           | 1.310           | 0.700                | 0.680                | -0.960               |

Note, that the ambient turbulence must be entered in percent (i.e. 10) when using the Quarton-Ainslie constants, while the TNO-constants are with ambient turbulence as decimal number (i.e. 0.10).

### References

- [1] Quarton and Ainslie, *Turbulence in Wind Turbine Wakes*, Wind Engineering Vol 14 No 1  
 [2] *European Wind Turbine Standards II*, ECN-C--98-096, December 1998.

## 10. Turbulence Model – B. Lange

The B. Lange turbulence model can only be used with the eddy viscosity wake model, because the turbulence parameters are derived directly from the eddy viscosity.

### Turbulence within the Wake

The turbulence intensity,  $I_T$ , is defined as the standard deviation of the wind speed process divided by the mean wind speed, i.e.

$$I_T = \sigma_u / u_0$$

It is possible to relate the eddy viscosity to the turbulence intensity. According to Lange et al [1], the turbulence intensity within the wake can be calculated using the following relation below. Please note that the equation relates to the free wind speed,  $U_0$ :

$$I_T = \varepsilon \frac{2.4}{\kappa \cdot U_0 \cdot z_h}$$

### Alternative Empirical Approach

Another alternative empirical characterization of the wake turbulence was proposed by Quarton and Ainslie [2]. Their equation is based on a parameterization on the near wake length – which is primarily used in relation with the Eddy Viscosity model. They report, that the empirical turbulence decay is somewhat higher than other model predictions. The equation is:

$$I_{add} = 4.8 C_T^{0.7} I_{amb}^{0.68} [X / X_n]^{-0.57}$$

where  $I_{add}$  is the added turbulence intensity from the wind turbine wake  
 $I_{amb}$  is the ambient wind speed  
 $X$  is the downstream distance  
 $X_n$  is the near wake length

This alternative approach can also be used with other wake modes, as the near wake length is easily determined through empirical equations. For further details on the near wake length – please see the chapter on the Eddy viscosity wake model.

### References

- [1] Lange, Bernard; H.P. Waldl; A.G. Guerrero; D. Heinemann & R.J. Barthelmie: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003 6:87-104.
- [2] Quarton & Ainslie: *Turbulence in Wind Turbine Wakes*, Wind Engineering, Volume 14, No. 1.

## **11. Turbulence Model – G.C. Larsen**

The G.C. Larsen is a simple empirical equation to determine the turbulence level within the wake. Reference is made to the paper '*Wind Field in Wakes*' [1] and the European Research Project – European Wind Turbine Standards - EWTS II [2].

### **Turbulence Intensity**

At positions downstream of the turbine, the wake added turbulence intensity can be determined from the equation:

$$I_w = 0.29S^{-1/3} \sqrt{1 - \sqrt{1 - C_T}} \quad (1)$$

where  $S$  is spacing expressed in rotor diameters  
 $C_T$  is the thrust coefficient

The expression for turbulence intensity is only valid for distances larger than two rotor diameters downstream.

### **References**

- [1] G.C.Larsen, J. Højstrup, H.A. Madsen, *Wind Fields in Wakes*, EUWEC '96, Gothenburg, 1996.
- [2] *European Wind Turbine Standards II*, ECN-C-99-073, 1999

**ANNEX A:**

**Case: Klim Fjordholme – Denmark**

This site holds a medium sized wind farm. Even if the site is inland, then it is situated close open water at the Limfjorden. The project: Turbines at this site are situated in the north-western part of Denmark. It is a good wind location with a mean wind speed equal to approximately 7 m/s given at 44 m a.g.l. (hub height). The park contains 35 WTGs, all Vestas V44, 600 kW, with typical spacing: 4.5 RD in rows and 5.5 – 7 between rows. The first 13 turbines were erected in September 1996 - one year after the wind farm was completed to 35 WTGs. The wind data are taken a 10 meter mast at Silstrup – situated approximately 30 kilometers west of this site. The wind data is sorted, so that calm periods are taken out.

For a detailed site analysis and additional data – the reference [1] should be consulted.



**Classification of wake model parameters**

Since the terrain is flat and open, wake calculations have been made with different terrain classifications in order to test which fits the measured data the best. Below is shown the main model parameters used in the different roughness classifications. Please note, the classification applies to the wake model only. The flow model (WAsP) uses a separate roughness classification.

|                    | Roughness class | Wake decay constant | Ambient turbulence intensity |
|--------------------|-----------------|---------------------|------------------------------|
| Mixed water/ land  | RC 0.5          | WDC=0.052           | TI=7.2%                      |
| Very open farmland | RC 1.0          | WDC=0.063           | TI=9.7%                      |
| Open farmland      | RC 1.5          | WDC=0.075           | TI=10.6%                     |
| Mixed Farmland     | RC 2.0          | WDC: 0.083          | TI=11.5%                     |

**Wind Farm Energy**

The different wake calculation methods have been tested on the Klim Fjordholmene site. The result is shown in the figure below.

## Case Studies – Wake Loss Calculations

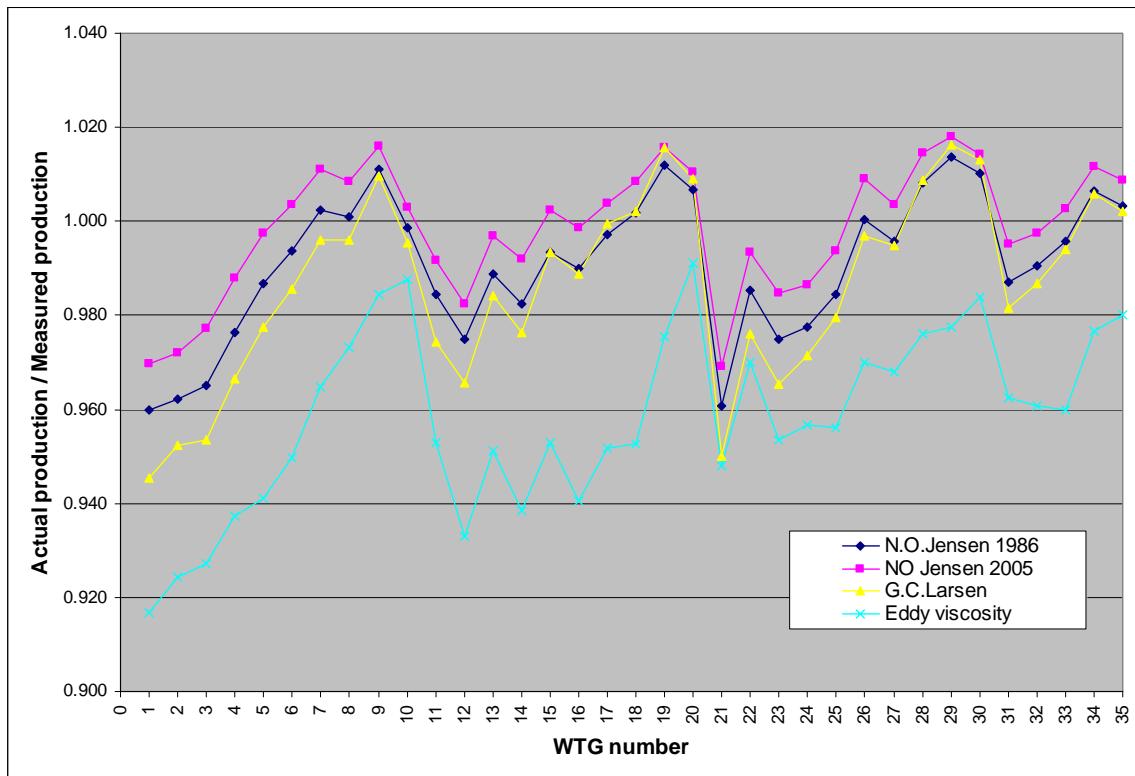
| Model name            | Park model – classification of terrain |                    |               |                |
|-----------------------|--|--------------------|---------------|----------------|
|                       | Mixed water/ land                      | Very open farmland | Open farmland | Mixed farmland |
| N.O. Jensen (1986)    | 71945 MWh                              | 72831 MWh          | 73633 MWh     | 74080 MWh      |
| N.O. Jensen (2005)    | 72809 MWh                              | 73560 MWh          | 74255 MWh     | 74654 MWh      |
| G.C.Larsen (1996)     | 71441 MWh                              | 72916 MWh          | 73320 MWh     | 73752 MWh      |
| Eddy Viscosity (2005) | 69582 MWh                              | 70773 MWh          | 71117 MWh     | 71671 MWh      |
| Measured energy       | 74767 MWh                              |                    |               |                |
| Energy (no wakes)     | 80900 MWh                              |                    |               |                |

As seen from the calculations above, the ‘mixed farmland’ classification seems to match the measured wake data for the class of N.O. Jensen wake models. However, the G.C. Larsen model and especially the Eddy Viscosity wake model seem to over estimate the wake losses quite a bit.

The reason for this over-estimation could be that neither of the wake models do include a model for wake meandering. Also, the used wake combination model (the squared wake deficit model), may not be appropriate for these models – since they both reflect a ‘bell shaped’ wake profile.

### Results from single WTGs

In the figure below is shown the ratio between measured and modelled energy yield. As seen, there is a significant bias in all models for the all downwind turbines. The calculation was made with  $TI=11.5\%$  and  $WDC=0.083$ . The difference in model performance is believed to arise from the (slight) differences in the wake combination method (squared velocity deficit method). This issue is to be investigated further.



### References

[1] P. Nielsen et al, *20 Detailed Case Studies*, EMD.dk A/S, December 2002, available online as pdf-files <http://www.emd.dk> -> go to ‘reports’