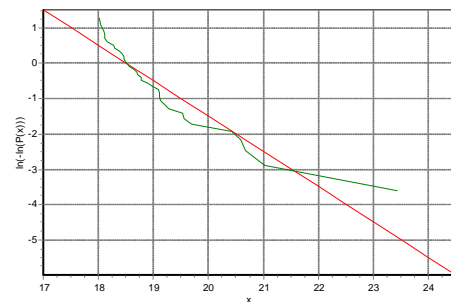
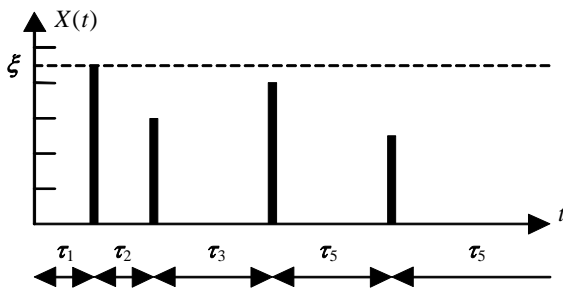
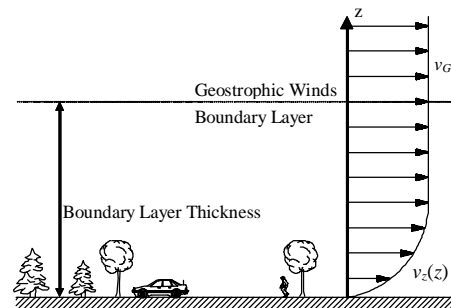


WindPRO / PARK

Introduction to the Estimation of Extreme Wind Speeds and Wind Loads



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Front cover

The front cover shows WindPRO logo, an idealized logarithmic wind profile, a Gumbel plot and an idealized Poisson process.

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1. Introduction

This document gives an introduction to the estimation of extreme wind speeds.

This document focuses on the extreme wind analysis by the means of a statistical extreme value analysis of time series.

An alternative approach (and recommended approach) is to use national structural codes; however this approach is not the primary subject of this document.



Figure 1: Semi-offshore turbines at Frederikshavn, Denmark.

2. Extreme Winds - Using Structural Codes

The extreme wind speeds for a given site are typically given in structural codes – used nationally in order to determine the safety of structures. The extreme wind speed – such as the 50 year extreme event – is then typically given in standardized conditions – e.g. by specifying the 10 minute mean wind speed and a corresponding roughness length (or roughness class). The structural engineer can then perform an on-site roughness classification – and then recalculate the standardized conditions to the on-site conditions.

Procedures for determining the extreme wind speeds using the structural codes are described in many codes e.g. the *Eurocode 1 – Actions on Structures* [1] with the national application documents - or the *Danish Code of Practice for Loads for the Design of Structures DS 410* [2], see below.

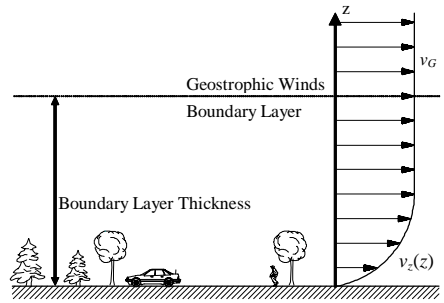
Danish Code of Practice – DS 410:1982

The following text (extracted from [3]) outlines the specification of extreme wind loads as found in the DS 410:1982.

The mean wind velocity, $v(z)$, can be described using the logarithmic velocity profile [4]:

$$v(z) = \frac{1}{\kappa} v_F \ln\left(\frac{z}{z_0}\right) = \frac{1}{0.4} \left(\sqrt{\frac{\tau_0}{\rho}} \right) \ln\left(\frac{z}{z_0}\right)$$

where κ is the Kármán's universal constant, $\kappa=0.4$
 v_F is the friction velocity
 z_0 is the surface roughness
 τ_0 is the friction between air and ground
 ρ is the density of air



Principle of the Velocity Profil([from[3])

The Danish code of Practice, DS 410, uses the velocity profile above in a slightly different form. In this, the equation is written:

$$v(z) = v_b k_t \ln\left(\frac{z}{z_0}\right)$$

where v_b is the basis wind velocity, determined mean wind velocity in 10 meters height
 k_t is a terrain parameter

The basis wind velocity is determined as the 50 year event 10 minutes mean wind velocity in 10 meters height with a surface roughness $z_0=0.05$. By comparison the equations above, it is easily seen the terrain parameter k_t has to comply with

$$k_t = \frac{v_F}{\kappa \cdot v_{z_0=0.05}(10)}$$

DS 410 states the terrain parameters for 3 different surface roughness: $z_0=0.01$, $z_0=0.05$ and $z_0=0.3$. The mean wind velocities are then obtained for these different terrain-types. (See table below)

2. Extreme Winds – Using Structural Codes

Terrain Type, z_0	Terrain parameter, k_t	Mean Speed $v(10)$
0.01	0.17	31.7
0.05	0.19	27.0
0.30	0.22	20.8

The 50 Year Event of the Mean Wind Speed, according to DS 410:1982 [2].

References:

- [1] *Eurocode 1 : Actions on structures – (1-4 General actions - Wind actions).*
- [2] *Dansk Ingeniørforenings's Code of Practice for Loads for the Design of Structures*, DS410, 3rd edition, Dansk Standard DS 410 (or later edition)
- [3] Misfeldt & Thøgersen, *Reliability Analysis of Wood Structures*, Aalborg University, 1997 (unpublished)
- [4] Michael Brorsen: *Vind og vindbelastning*, Forelæsningsnoter til kursus i Strømningslære, Aalborg Universitet, 1996 (*Lecture Notes – in Danish - Wind and Windloads*)

3. Extremes - Using Probabilistic Methods

The extreme wind speeds may – alternatively - be determined through a long time series – available near the site of interest. The time series should last at least 10 years – otherwise the uncertainty in the estimation will be too large. Also, all significant extreme climatic events should occur in the time series – e.g. tropical cyclones. Beware, that no climatic event has so long return periods that they are not represented in the time series, e.g. tropical locations with very rare events of tropical cyclones. In those cases, other methods should be used – e.g. in lack of better data - climatic simulations using meso scale models could be used.

Introduction

The Gumbel distribution is widely used for determining extreme (extrapolated) values of measured distributions and time series data. The measured data is typically fitted to the Gumbel distribution – then the 50-year extreme value is determined (or another return period). Additionally, the uncertainty may be determined through a bootstrap resembling technique.

About the Gumbel Distribution (from [3])

The Gumbel distribution is also known as the Extreme Type I distribution of the largest extreme. This distribution can be applied if the upper tail of the parent distribution falls off in an exponential manner. The Gumbel distribution is a two-parameter distribution with the parameters α and u . The distribution function of some random variable X is:

$$F_X(x) = \exp[-\exp[-\alpha(x-u)]]$$

Where $\alpha > 0$ and $-\infty < x < \infty$. The mean value and standard deviation are given by:

$$\mu_X = u + \frac{\gamma}{\alpha} \quad \sigma_X = \frac{\pi}{\sqrt{6} \cdot \alpha}$$

where γ is Euler's constant and equal to 0.5772. A useful property of the Gumbel distribution is, that the distribution of the largest extreme in n independent trials of a random variable, X , having a distribution function following (A.18) is:

$$F_{X_n}(x) = \exp[-n \cdot \exp[-\alpha(x-u)]]$$

the standard deviation of X_n does not change from the σ_X , given above, but the mean value can be calculated from:

$$\mu_{X_n} = \mu_X + \frac{\sqrt{6}}{\pi} \sigma_X \ln(n)$$

Defining the extreme event through a characteristic value

A characteristic event (load) is typically determined as the 98% percentile in the annual extreme event (load) distribution, $F_X(x)$. Thus, the characteristic event is determined as

$$x_k = F_X^{-1}(0.98)$$

3. Extreme Winds – Using Probabilistic Methods

The characteristic event – determined from the 98% percentile – is an event that is only exceeded with an annual probability of 2%. It is obvious to define this event as a 50 year event, as we per definition set the relation between return period - T – and the percentile - f_T – in the annual extreme value distribution as:

$$f_T = 1 - \frac{1}{T}$$

The above equation is not useable for $T \leq 1$, as the percentile here becomes less than or equal to 0.

Example (from [5]) The annual extreme value distribution of a load, P , follows a Gumbel distribution. The mean value of this extreme load is $\mu=20$ kN and the standard deviation is $\sigma=8$ kN (these values corresponds to a coefficient of variation equal to $COV=\sigma/\mu=0.40=40\%$). The 50 year extreme event is now determined from the 98% percentile of the extreme value distribution. First, the distribution parameters α and β are determined:

$$\alpha = \frac{\pi}{\sqrt{6}\sigma} = \frac{\pi}{\sqrt{6} \cdot 8} = 0.160 \quad \beta = \mu - \frac{\gamma}{\alpha} = \mu - \frac{0.5772}{\alpha} = 20 - \frac{0.5772}{0.160} = 16.4$$

The characteristic load is now determined as the 98% percentile by inverting the cumulative distribution equation:

$$p_k = \beta - \frac{1}{\alpha} \ln[-\ln[F_p(p)]] = 16.4 - \frac{1}{0.160} \ln[-\ln[0.98]] = 40.7 \text{ kN}$$

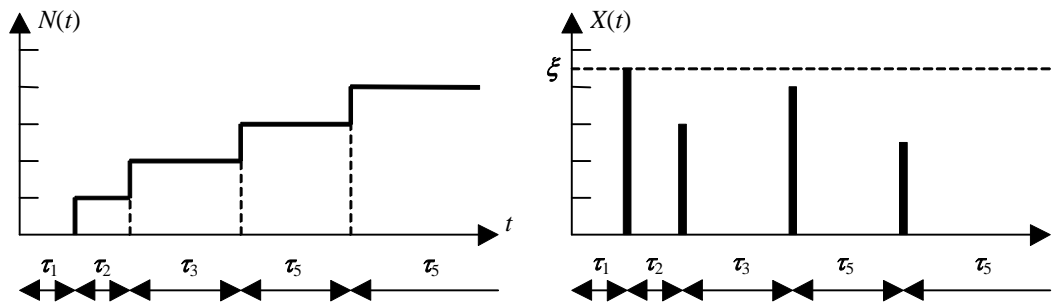
As shown, the characteristic value is – in this case – approximately the mean value plus 2.5 times the standard deviation.

Defining the return period through a stationary Poisson process

An alternative definition of the T -year extreme event is found by considering a stationary Poisson-process. The Poisson-process is a stochastic counting process, characterized by the intensity ν (the intensity is the number of events per unit time). It is known (see [6]), that the inter-arrival times (the time between two events) in a Poisson-process follow an exponential distribution. Then the return period, R , is determined as the expected value of the inter-arrival times of the Poisson process. The expected value and the variance of the inter-arrival times are

$$E[T] = \frac{1}{\nu} \quad \text{Var}[T] = \frac{1}{\nu^2}$$

An example of a Poisson process is shown below



Left figure: Poisson process with inter-arrival times, τ . Right figure: Poisson spike-process.

3. Extreme Winds – Using Probabilistic Methods

A distribution function describing the ‘height’ of the spike in the Poisson process is now added to each event of the Poisson-process (as shown in the figure above – right side). It is then possible to derive the maximal level ξ , which is present in the period $[0;T]$. This level has the following distribution (presumed the Poisson-process):

$$F_{\max}(\xi) = \exp\left[-\nu \cdot T \cdot (1 - F_X(\xi))\right]$$

If a fixed level ξ is set, the process consisting of crossings of this new level is also a new Poisson process with the intensity:

$$\nu_{\xi} = \nu \cdot (1 - F_X(\xi))$$

The expected inter-arrival time for this new Poisson process is:

$$E[T] = \frac{1}{\nu_{\xi}} = \frac{1}{\nu \cdot (1 - F_X(\xi))}$$

The return period for the extreme event is now defined from the expected time between two crossings of the level ξ . The n -year return period is defined as $n=E[T]$. I.e. from the equations above there is a relationship between probability of crossing the level and the return period

$$F_{\max}(\xi) = \exp\left[-\frac{T}{E[T]}\right] \Leftrightarrow E[T] = -\frac{T}{\ln[F_{\max}(\xi)]}$$

The equation above – left side – approaches $1/n$ for large numbers of n . This corresponds to the definition made earlier from the raw percentiles. By the way, the T-year period is typically equal to 1 year.

Example (from [5]) We want to determine the annual probability of exceeding the 50-year event. Using the equation above:

$$F_{\max}(\xi) = \exp\left[-\frac{T}{E[T]}\right] = \exp\left[-\frac{1}{50}\right] = 0.9802$$

I.e., the annual probability of exceeding the 50-year event is 1.98%. Assuming the same load as in the previous example, the extreme load is now determined to 40.8 kN.

The DS 410 Design Wind Velocity and the Extreme Distribution

The Danish code of Practice of Loads, DS 410, uses the 50-year event as the deterministic design-value. This value is determined from the 1-year extreme event, using probability of non exceedence of 0.98, i.e. $F_V(v)=0.98$. No distribution function is stated in the code for the wind load, but some authors e.g. Dyrbye et al [7] and Hansen [8] state that the annual extreme-mean wind (velocity) distribution can be described by an extreme-type I (Gumbel) distribution, see above. With a surface roughness of $z_0 = 0.05$, ν is found from statistical investigations to be 27 m/s. This value of the 50-year event is used in DS 410.

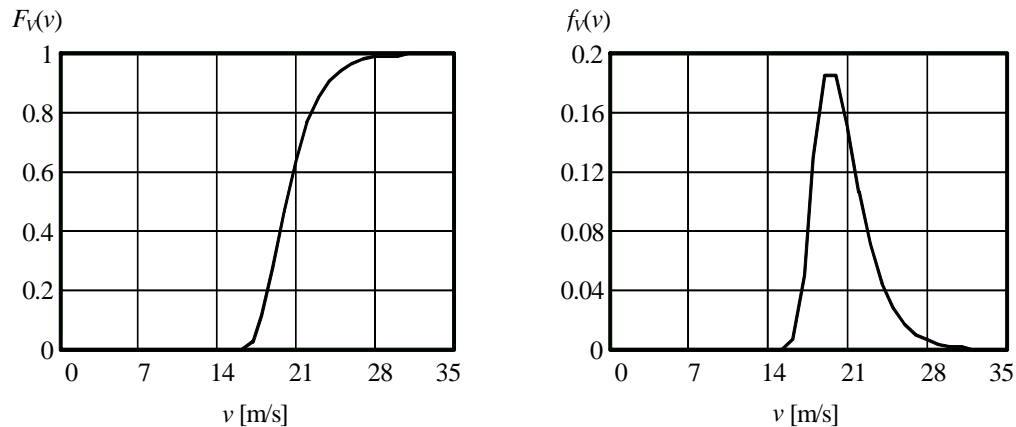
3. Extreme Winds – Using Probabilistic Methods

If this extreme distribution is used in the European area, then the coefficient of variation is approximately $COV_W=0.12$ (Dyrbye et al [7]). This value is in agreement with measurements performed in the Danish area, see e.g. Hansen [8].

The extreme distribution for the annual wind velocity based on the current code prescriptions is found to be:

$$F_{V_{DS}}(v_{DS}) = \exp\left[-\exp\left[-0.51899(v_{DS} - 19.481)\right]\right]$$

The distribution and density functions matching the equation above are drawn in the figure below:



The Annual Extreme Wind Distribution Based on Code Specifications in DS 410:1982 [2].

Gumbel Distribution Fitting

When using the WindPRO extreme wind speed estimation tool, then distribution parameters are estimated using least-squares estimates. Reference is made to S. Ross [10]

Statistical Uncertainties

The statistical uncertainties in extreme estimations may be (partly) determined through a bootstrap re-sampling technique (as in the WindPRO extreme wind speed estimation tool). Reference is made to Efron and Tibshirani [9].

References

- [1] Eurocode 1: Actions on structures – (1-4 General actions - Wind actions).
- [2] Dansk Ingeniørforenings's Code of Practice for Loads for the Design of Structures, DS410, 3rd edition, Dansk Standard DS 410 (or later edition)
- [3] Misfeldt & Thøgersen, *Reliability Analysis of Wood Structures*, Aalborg University, 1997 (unpublished)
- [4] Michael Brorsen: *Vind og vindbelastning*, Forelæsningsnoter til kursus i Strømningslære, Aalborg Universitet, 1996 (*Lecture Notes – in Dahish - Wind and Wind Loading*)
- [5] Morten Lybech Thøgersen, *Notat vedr. ændring af lastens returperiode*, Risø National Laboratory, 1999, (available online through www.risoe.dk/vea/offdes).
- [6] H.O.Madsen, S. Krenk & N.C. Lind: *Methods of Structural Safety*, Pentice-Hall, Englewood Cliffs
- [7] C. Dyrbye, S. Graversen, S. Krenk, N.C. Link and H.O. Madsen: *Konstruktioners sikkerhed*, Den private Ingeniørfond ved Danmarks tekniske Højskole, 1979.

3. Extreme Winds – Using Probabilistic Methods

- [8] Svend Ole Hansen: *Reliability of Wind Loading on Low-Rise Buildings in a Group*, Afdelingen for Bærende Konstruktioner, DTH, 1992.
- [9] Efron & Tibshirani: *An Introduction to the Bootstrap*, Monographs on Statistics and Applied Probability, Chapman & Hall, New York,
- [10] S.M. Ross: *Introduction to probability and statistics for engineers and scientists*, San Diego, Harcourt/Academic, 2000

4. Case Study – Klim Fjordholme – Denmark

The terrain is flat and open. Even if the site is inland, then it is situated close open water at the Limfjorden. The project: Turbines at this site are situated in the northwestern part of Denmark. It is a good wind location with a mean wind speed equal to approximately 7 m/s given at 44 m a.g.l. (Hub height). The park contains 35 WTGs, all Vestas V44, 600 kW, with typical spacing: 4.5 RD in rows and 5.5 – 7 between rows. The first 13 turbines were erected in September 1996 - one year after the wind farm was completed to 35 WTGs.

For a detailed site analysis – the reference [1] should be consulted.

Wind Data for the Extreme Wind Analysis

In lack of real measured data, the 20 years time series data is taken from the NCAR/NCEP reanalysis project – holding global wind data for every 2.5 degrees. We use the 10 m data at the position N 57.5 E 10.0. The data were downloaded using the EMD online wind data service available in WindPRO 2.5.



The fitting method used is a least squares method. We calculate the extreme as omnidirectional distribution only, however the tool in WindPRO opens for doing directionally dependent estimates. The uncertainty (UNC) is the statistical uncertainty calculated using a bootstrap sampling technique. The UNC is taken as one standard deviation.

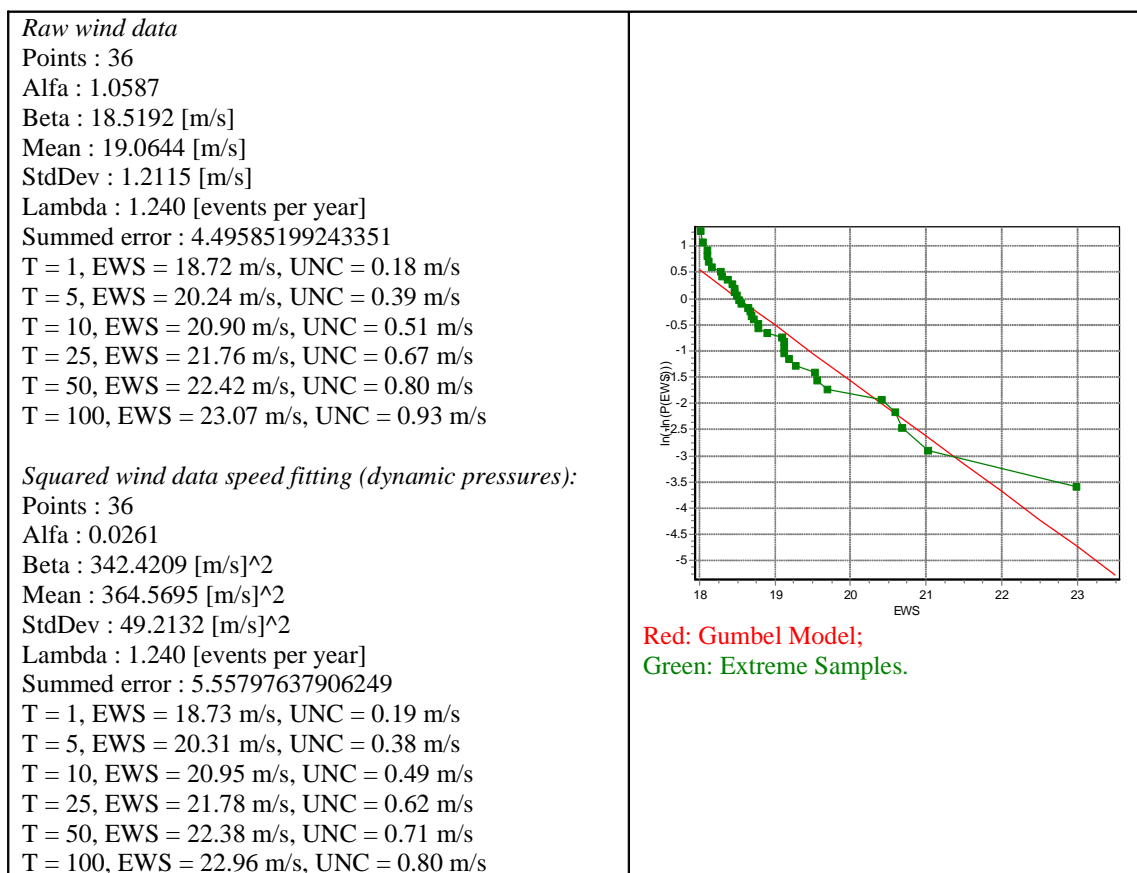
NCEP / NCAR Data at 10 meters height

Data overview:

First record dated : 01-01-1976
Last record dated : 31-07-2005 18:00:00
Sample rate : 360 minutes
Number of samples (enabled data only): 43220
Max wind speed: 22.99
Mean wind speed: 6.45
Height : 10

The threshold value is adjusted until approximately 50 extreme values are left. The threshold value is determined to 18 m/s – leaving 36 independent extremes. The Gumbel data fitted to this data is shown in the table below.

Case Studies – Extreme Wind Speeds

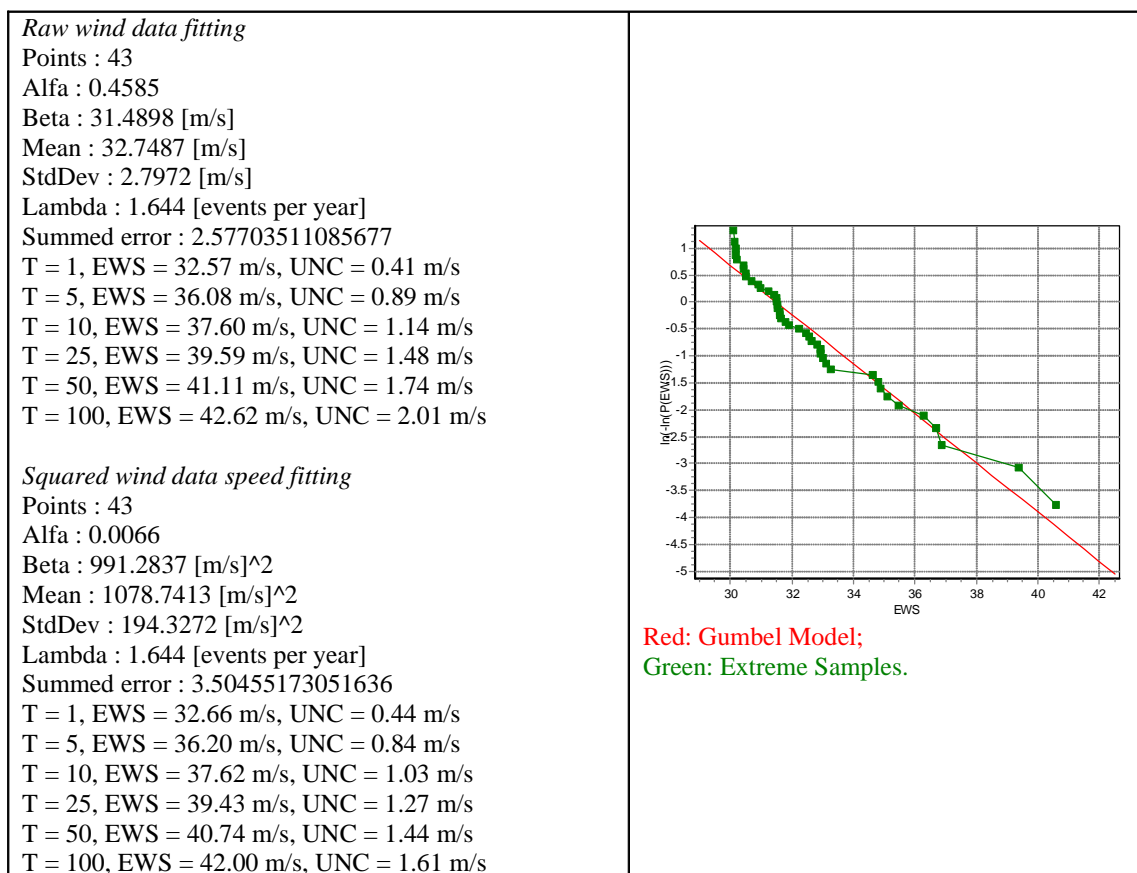


Upper air NCEP / NCAR Data (850 hPa pressure level data)

Data overview:

First record dated : 01-01-1976
 Last record dated : 31-07-2005 18:00:00
 Sample rate : 360 minutes
 Number of samples (enabled data only): 43220
 Max wind speed: 40.59
 Mean wind speed: 9.82
 Height : 850 hPa (upper air data)

The threshold value is adjusted until approximately 50 extreme values are left. The threshold value is determined to 30 m/s – leaving 43 independent extremes. The Gumbel data fitted to this data are:



Comparing the results

The upper air data gives an estimate of the 50-year geotropic wind equal to 40.74 m/s or 41.11 m/s. The NCAR 10 m data gives an estimate of the 50 year 10 m wind equal to = 22.42 m/s or 22.38 m/s. The Danish code of practice (see chapter 2) – estimates the extreme wind to either 24 m/s or 27 m/s using standard conditions (10 m height, $z_0=0.05$ m). The higher value is used for sites near to the west coast.

H. Frank [2] estimates the 50 year wind speed from NCAR 1948-1999 data. The 10 m extreme wind speed is estimated to approximately 20 m/s. However, this reference uses a different fitting procedure (moment weighted procedure) – which is believed to yield slightly different results. However, the statistical uncertainty to the NCAR data estimates is in the magnitude of 1.0-2.0 m/s so the results may be regarded as identical.

Case Study – Validation

In order to validate the method, a calculation is made with an artificial generated time series. In this time series, the annual extreme values are sampled from a Gumbel distribution with the mean value equal to 40.0 m/s and a coefficient of variation equal to 20%. A 50 year long time series is generated with these extremes, filling the missing days with wind speeds sampled from a Gaussian distribution with mean 10.0 m/s and coefficient of variation equal to 20%. The 50 year event of this distribution – taken as the 0.98 percentile of the distribution - is 60.80 m/s, or approximately equal to the mean value plus 2.5 times the standard deviation.

The 50 extremes of the time series are shown below. The mean value of the 50 samples is 38.7 m/s and the coefficient of variation is 20.1%. A random seed equal to 132 was used to generated this series.

Case Studies – Extreme Wind Speeds

32.90	34.78	55.00	35.12	34.63	34.91	49.45	36.39	47.75	38.60	27.31	31.77
36.88	30.14	39.20	36.58	30.09	32.12	44.21	54.84	33.20	51.57	38.89	32.48
31.09	33.03	42.54	32.30	40.13	58.26	51.52	40.49	44.65	47.39	37.44	39.32
28.95	37.85	53.26	32.07	51.82	33.73	33.33	31.40	40.17	38.96	36.32	32.16
32.43	37.64										

Based on the distribution parameters from the Gumbel distribution, the expected maxima using different return periods can be calculated. This is shown in the table below, as well as the estimates calculating the WindPRO tool. Note, that the expected maximum of T-independent trials of the annual extreme Gumbel distribution gives a higher result than if the 'percentile' definition is used. The uncertainty from the sample estimates are given as \pm one standard deviation.

T-Period [years]	Extreme wind speed from Gumbel Distribution or Samples				
	Expected max of T trials [m/s]	Characteristic WSP from percentile [m/s]	50 Samples (seed 132) [m/s]	50 Samples (seed 100) [m/s]	1000 Samples (seed 125) [m/s]
1	40.00	36.40	35.02 \pm 1.05	33.90 \pm 0.87	36.30 \pm 0.22
5	50.04	46.44	45.94 \pm 2.09	42.97 \pm 1.74	46.69 \pm 0.44
10	54.36	50.76	50.64 \pm 2.80	46.88 \pm 2.33	51.17 \pm 0.57
25	60.08	56.48	56.85 \pm 3.79	52.07 \pm 3.15	57.09 \pm 0.76
50	64.40	60.80	61.56 \pm 4.56	55.95 \pm 3.79	61.56 \pm 0.91
100	68.73	65.12	66.26 \pm 5.33	63.76 \pm 5.08	66.04 \pm 1.06
1000	83.09	79.49	-	-	-

Data overview:

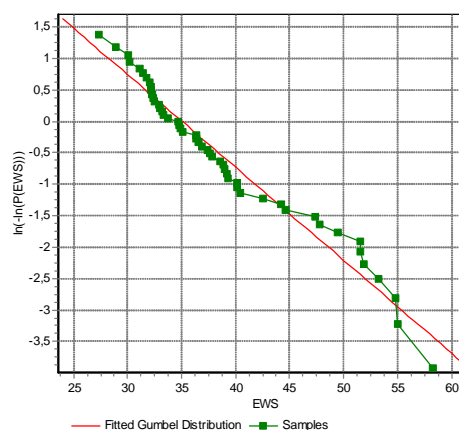
First record dated: 01-01-1900 01:00:00
 Last record dated: 31-12-1949 01:00:00
 Sample rate: 1440 minutes
 Number of samples (enabled data only): 18262
 Max wind speed: 58,26
 Mean wind speed: 10,07
 Height : 132 (is actually set as the random seed)

The threshold value is determined to 22 m/s – leaving 50 independent extremes. The Gumbel data fitted to this data as shown in the table below (random seed 132).

Raw wind data fitting

Points : 50
 Alfa : 0,1474
 Beta : 35,0205 [m/s]
 Mean : 38,9357 [m/s]
 StdDev : 8,6993 [m/s]
 Lambda : 1,000 [events per year]
 Summed error : 2,24036021517607
 T = 1, EWS = 35,02 m/s, UNC = 1,05 m/s
 T = 5, EWS = 45,94 m/s, UNC = 2,09 m/s
 T = 10, EWS = 50,64 m/s, UNC = 2,80 m/s
 T = 25, EWS = 56,85 m/s, UNC = 3,79 m/s
 T = 50, EWS = 61,56 m/s, UNC = 4,56 m/s
 T = 100, EWS = 66,26 m/s, UNC = 5,33 m/s
 T = 200, EWS = 70,96 m/s, UNC = 6,11 m/s

Squared wind data speed fitting



Red: Gumbel Model;
 Green: Extreme Samples.

Points : 50 Alfa : 0,0017 Beta : 1242,4716 [m/s]^2 Mean : 1577,4365 [m/s]^2 StdDev : 744,2784 [m/s]^2 Lambda : 1,000 [events per year] Summed error : 3,43466655530497 T = 1, EWS = 35,25 m/s, UNC = 1,29 m/s T = 5, EWS = 46,65 m/s, UNC = 1,87 m/s T = 10, EWS = 50,78 m/s, UNC = 2,29 m/s T = 25, EWS = 55,77 m/s, UNC = 2,80 m/s T = 50, EWS = 59,27 m/s, UNC = 3,16 m/s T = 100, EWS = 62,57 m/s, UNC = 3,49 m/s T = 200, EWS = 65,71 m/s, UNC = 3,79 m/s	
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References

- [1] P. Nielsen et al, *20 Detailed Case Studies*, EMD.dk A/S, December 2002, available online as pdf-files <http://www.emd.dk> -> go to 'reports'
- [2] H. Frank: *Extreme Winds over Denmark from the NCEP/NCAR Reanalysis*, Risø-R-1238(EN), Risø National Laboratory, 2001,
- [3] P. Thoft-Christensen & M. J. Baker: *Structural Reliability Theory and Its Applications*, Springer Verlag, 1982

5. User Guide

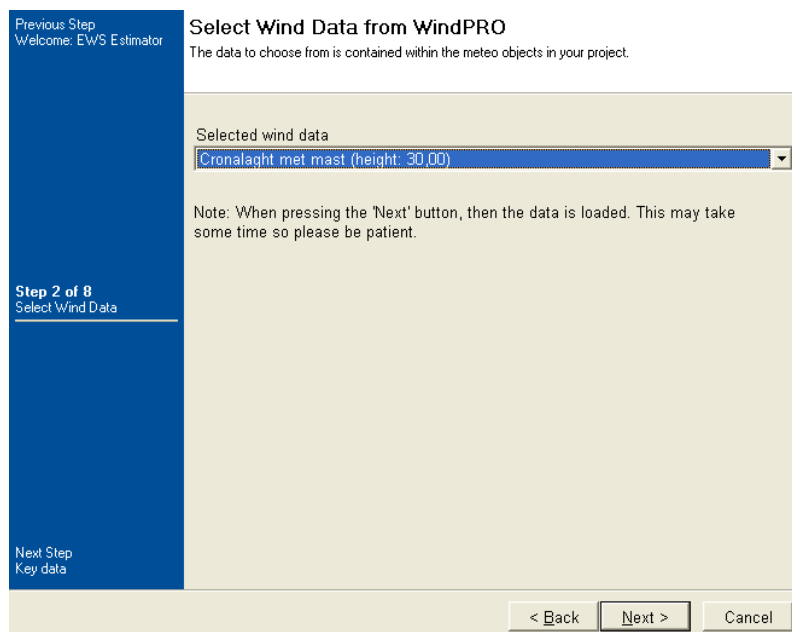
This chapter gives instructions on how to operate the Extreme wind speed estimator in WindPRO. The theoretical background is covered in the previous two chapters.

Background

The Extreme wind speed estimator can be started from the Tools menu in Maps and objects. It works as a wizard that guides the user through each step of the process. The estimator requires a meteo object in the project with a data time series. While it can work on time series of any length, longer time series will result in more reliable results. Preferably a time series should hold at least 10 years of measurements. Also the calculation will give extreme wind speeds with the averaging time which has been used as input data, i.e. a time series of hourly mean values will give an extreme hourly mean value. A time series of gust values will give the extreme gust wind speed.

Selection of data

Starting the estimator will bring you to the opening window of the estimator wizard. Continue to the next window by clicking “Next”.



Here it is possible to select among the available time series. Each height on each Meteo object in the project can provide a time series.

Key data

At step 3 the key figures of the selected time series are presented. These include the measurement period, sample rate and how many samples have been included. Mean and max values are found. These data can be saved to a text file by pressing the “Save data to file” button.

Key data

This page contains some key data from your selected meteo object. Please verify that the data series is suitable for extreme value analysis.

Descriptive data from the selected measurement

Cronalaght met mast (height: 30,00)

Descriptive data from the selected measurement

First record dated : 14-01-1994 18:30:00
 Last record dated : 09-04-1996 19:10:00
 Sample rate : 10 minutes
 Number of samples (enabled data only): 111806
 Max wind speed: 34,36
 Mean wind speed: 9,31
 Height : 30

Save data to file

< Back Next > Cancel

Press Next.

Extraction parameters.

At step four the extraction parameters must be defined. The analysis requires only one measurement from each independent episode. There must be so many selected measurements as to provide a sufficiently reliable fit, but not so many as to include too many low wind speed events that will affect the information on the high speed events. Between 20 and 50 m/s is recommended.

This is done by selecting a threshold value, so only values higher than this will be included and a independence criteria making sure only one measurement from each storm episode is included. For each episode the highest value is chosen.

While 3 to 4 days seem to be a reasonable independence criterion it is not so easy to recommend a threshold value. Usually testing different wind speeds will be needed in order to establish which wind speed will provide the necessary number of samples.

The Sector to and Sector from options makes it possible to test extreme wind speeds for particular sectors.

5. User Guide

Previous Step
Key data

Step 4 of 8
Extraction parameters

Next Step
Estimation method

Extraction parameters

Setup the parameters for determining a sample extreme distribution from your selected data.

Threshold value [m/s] - data below this value is removed

Independence criteria [days]

Sector from [deg]

Sector to [deg]

< Back Next > Cancel

Choose method

At step 5 two methods can be chosen: Either to calculate using the real wind speed or by using the dynamic pressure (squared wind speed). The theoretical background covers the real wind speed choice. Dynamic pressure is an alternative to this.

Previous Step
Extraction parameters

Step 5 of 8
Estimation method

Next Step
Data removed

Choose method

In order to estimate the extreme wind speeds, two methods has been implemented. One uses the raw wind data, the other analyses on the squared of the wind speed (dynamic pressures).

Choose method

☒ Real wind speeds (raw data)

☐ Squared wind speeds (dynamic pressures)

< Back Next > Cancel

Data removed – Results

The data which does not fulfil the independence and threshold criterion are now isolated and the resulting number of samples given. If the number of samples is below 20 or above 50 a warning is given and the threshold should be adjusted to reach the proper number of samples

This table can be saved to a text file.

Data Removed - Results

This page shows the number of removed points, the result for the extreme value analysis should be around 30-50 points left.

Data removed

Points before : 111806
Points removed (sectors) : 0
Points removed (threshold) : 109335
Points removed (independence) : 2424
Points after : 47

Save data to file

< Back Next > Cancel

Extreme distribution fitting

The samples are now fitted to a Gumbel function as described in the theory. The parameters of the fit is presented in the window and a graphic presentation of the fit can be obtained by clicking on the “Plot distribution” button..

An example of such a plot can be seen below.

The closer the data fit around the ideal Gumbel function the smaller the statistical uncertainty.

The plot can be copied to the clipboard.

5. User Guide

Previous Step
Data removed

Step 7 of 8
Extreme distribution

Next Step
Extreme Winds

Extreme distribution (Gumbel) fitting

Here the key results of the fitting procedure is found.

Gumbel distribution data (based on samples)

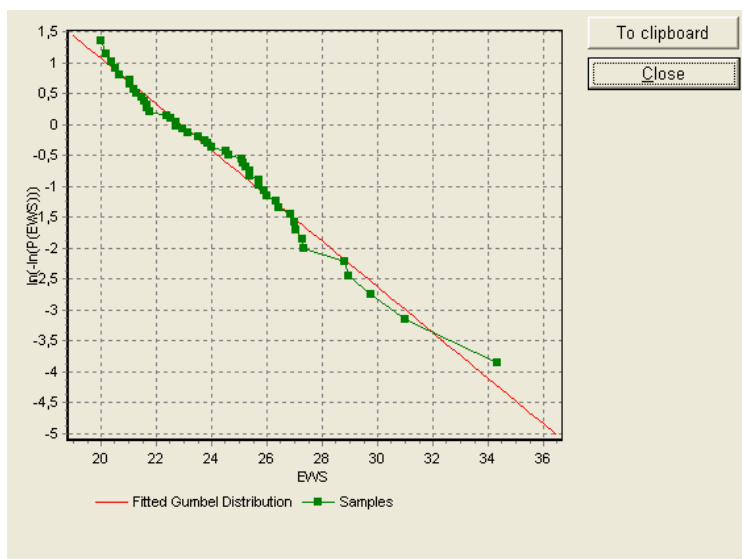
Estimation is based on the raw wind data.
Gumbel fitting is based on a least squares method

SAMPLE RESULTS
Mean [m/s]: 24,3874
StdDev [m/s]: 3,1320

FITTED RESULTS - SECTOR 0 DEGREES TO 360 DEGREES
Points : 47
Alfa : 0,3700
Beta : 22,9084 [m/s]
Mean : 24,4685 [m/s]
StdDev : 3,4664 [m/s]
Lambda : 21,037 [events per year]
Summed error : 0,985678633294886

Plot distributionCopy to Clipboard

< BackNext >Cancel



Estimated extreme wind speeds

Finally the extreme wind speed is calculated for different return periods based on the Gumbel fit. The statistical uncertainty relates to the Gumbel fit but does not include external uncertainties such as data quality and only to a limited degree the length of the time series.

The results can be copied to the clipboard.

5. User Guide

Previous Step
Extreme distribution

Step 8 of 8
Extreme Winds

Estimated Extreme Winds

Here, the estimated extreme winds are calculated based on a number of different return periods. The Extreme Wind Speed (EWS) is the expected maximum in the return period. Uncertainty (UNC) is given as one standard deviation.

Extreme winds for different return periods

Extreme estimation based on raw wind data
RESULTS - SECTOR 0 DEGREES TO 360 DEGREES
Samples : 47
T = 1, EWS = 31,14 m/s, UNC = 1,50 m/s
T = 5, EWS = 35,49 m/s, UNC = 2,23 m/s
T = 10, EWS = 37,37 m/s, UNC = 2,55 m/s
T = 25, EWS = 39,64 m/s, UNC = 2,98 m/s
T = 50, EWS = 41,71 m/s, UNC = 3,30 m/s
T = 100, EWS = 43,59 m/s, UNC = 3,62 m/s
T = 200, EWS = 45,46 m/s, UNC = 3,95 m/s
|

Copy to Clipboard

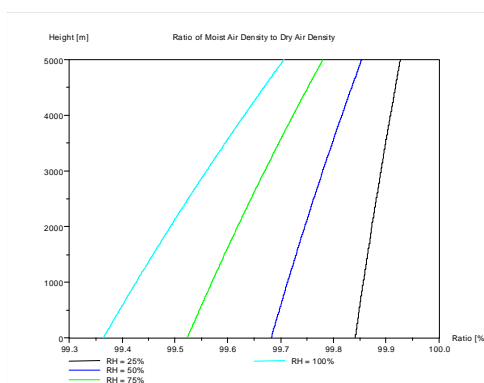
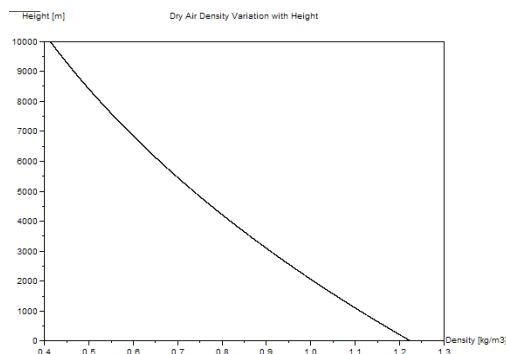
< Back

Finish

Cancel

WindPRO / ENERGY

Modelling of the Variation of Air Density with Altitude through Pressure, Humidity and Temperature



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Front cover

The front cover shows the air density variation with height – calculated using the U.S. Standard Atmosphere parameters. Also shown is the relative influence of moist air.

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1. Introduction

The air density calculations have been subjected to international standardization, presumably because the air density influences the lifting capacity of aircrafts. The air density varies with altitude and temperature. For wind turbines, the air density is a key parameter when estimating wind energy, as the energy output from the WTGs proportionally depends on this parameter. The estimated power output from the WTG, P , is given in the equation below.



$$P = 0.5 \cdot \rho \cdot w^3 \cdot A \cdot C_e \quad (1)$$

where P is the produced power output [W]

ρ is the air density [kg/m^3]

w is the wind speed [m/s]

A is the area swept by the rotor [m^2]

C_e is the total efficiency of the WTG at the given wind speed

While the energy calculations typically are calculations over a large time scale, the air density must be given as the expected mean density over the period considered. As an alternative to using a constant air density – which is reasonable assumption in most terrains - WindPRO offers a model to take the variations in air density with air temperature and pressure into account. This model could be used in mountainous terrains.

The model for the varying dry air density is mainly based on equations in the US Standard Atmosphere Model from 1976 [1]. Since moist air is less dense than dry air, also a model for including the variation in humidity is also implemented. This is based on equations from M. Salby [2].

The WindPRO model offers the option for the user to set site-specific temperatures, humidity and pressure parameters – thus enabling a detailed air density description for turbines situated with very varying hub-heights.

References

- [1] *U.S. Standard Atmosphere, 1976*, U.S. Government Printing Office, Washington, D.C.
- [2] Murry L. Salby: *Fundamentals of Atmospheric Physics*, Academic Press, 1996, Elsevier Science

2. Dry Air Density Variation with Altitude

This chapter gives an introduction to the physical and theoretical considerations needed when developing the set of equations describing the variation of dry air with altitude.



Cronalaght Wind Farm, Ireland.

Geopotential Altitude

Modelling of the standard atmosphere is typically done in terms of the geopotential altitude. This is due to a simplification of the equations describing the atmosphere. The idea behind the geopotential altitude is that a small change in geopotential altitude will cause the same change in gravitational potential energy as the same change in geometric altitude at sea level. This is mathematically expressed as:

$$g(Z)dZ = GdH \quad (1)$$

where $g(Z)$ is the acceleration of gravity (decreases at increasing altitudes)
 Z is the geopotential altitude
 G is the acceleration of gravity at sea level
 dH is the geometric altitude

The value of the gravity varies with height, and is found to follow the equation below

$$\frac{g(Z)}{G} = \left(\frac{E}{Z + E} \right)^2 \quad (2)$$

where E is the radius of the earth (~6535 km)

Combining the two equations above and integrating yields give an expression for the geopotential altitude.

$$\begin{aligned} dH &= \frac{g}{G} dZ = \left(\frac{E}{Z + E} \right)^2 dZ \\ H &= \int_0^H dH = \int_0^Z \left(\frac{E}{Z + E} \right)^2 dZ = \frac{EZ}{E + Z} \\ Z &= \frac{EH}{E + H} \end{aligned} \quad (3), (4), (5)$$

The relation between geopotential height and geometric height is used to calculate the table below. It is seen, that the difference between the two altitude measures is marginal for the lower levels of the atmosphere where wind turbines are situated. Thus, in WindPRO we approximate the geopotential height with the geometric height.

2. Dry Air Density Variation with Altitude

Geometric Height, Z [m]	Geopotential Height, H [m]	Difference %
100	99.99	0.002
500	499.96	0.008
1000	999.84	0.016
5000	4996.1	0.078
10000	9984.3	0.157
50000	49610	0.796
100000	98451	1.573

Table 1: Relation between geometric and geopotential height

The Perfect Gas Law

The air density may be estimated from the perfect gas law.

$$P \cdot V = n \cdot R_{air} \cdot T \quad (6)$$

where P is the pressure [Pa]
 V is the volume considered [m³]
 n is the number of moles
 R_{air} is the specific gas constant for dry air (287.05 J/(kg·K))
 T is the temperature [K]

If the density, ρ_{air} , is defined from the number of molecules in a certain volume, $\rho = n / V$, then the density may now be expressed from the perfect gas law as:

$$\rho_{air} = \frac{P}{R_{air} \cdot T} \quad (7)$$

The specific gas constant for dry air relates to the universal gas constant, R , and the mean molecular weight of air:

$$R_{air} = R / M_{air} \quad (8)$$

where R is the universal gas constant = 8.31432 J/(K·mol)
 M_{air} is 28.9644·10⁻³ (kg/mol) (assumed constant up to approximately 86 km altitude)

Temperature Variation in the Atmosphere

Within an atmospheric layer, the temperature variation is approximated as a linear function of the geopotential altitude.

$$T = T_b + L(H - H_b) \quad (9)$$

where L is the temperature lapse rate (temperature gradient) – which typically is negative for increasing heights
 T_b is the temperature at the base of the layer
 H_b is the geopotential altitude at the base level
 H is the geopotential height

2. Dry Air Density Variation with Altitude

The Hydrostatic Equation

Hydrostatic modelling of the atmosphere is a reasonable approximation – even if the atmosphere is in motion. This is because the vertical displacements of air and their time derivatives are small compared to the forces in the hydrostatic equation – see M. Salby [1]. The basic hydrostatic equation is

$$\begin{aligned} dP &= -\rho g(Z) dZ \\ &= -\rho G dH \end{aligned} \quad (10)$$

Air Density Calculations with Pressure Changes

Using the hydrostatic equation with the perfect gas law and the stepwise linear temperature variation assumption, the hydrostatic equation yield:

$$dP = -\frac{MG}{R_{air}} \frac{P}{(T_b + L(H - H_b))} dH \quad (11)$$

Integrating the equation gives the relation between base variables and the ones in the specified height [2]

$$\begin{aligned} \int_{P_b}^P \frac{1}{P} dP &= - \int_{H_b}^H \frac{MG}{R_{air}} \frac{1}{(T_b + L(H - H_b))} dH \\ \frac{P}{P_b} &= \exp \left[\frac{-GM(H - H_b)}{RT_b} \right] \quad \text{for } L = 0 \quad \text{and} \\ \frac{P}{P_b} &= \left[\frac{T_b + L(H - H_b)}{T_b} \right]^{\frac{GM}{RL}} \quad \text{for } L \neq 0 \end{aligned} \quad (12), (13), (14)$$

US Standard Atmosphere

The US Standard Atmosphere, published in 1986, holds a model for the development of pressure and density with altitude over the sea level. The model used in WindPRO is based on the specification for the lower 11 km of the atmosphere.

In the US Standard Atmosphere, seven fundamental layers are defined in the lower 86 kilometres of the atmosphere:

h_1 [km]	0	11	20	32	47	51	71
h_2 [km]	11	20	32	47	51	71	84.852
L (dT/dh) [K/km]	-6.5	0.0	1.0	2.8	0.0	-2.8	-2.0

The heights are given in geopotential heights. 84.852 km corresponds to a geometric height of 86 km. Standard values of other important parameters are:

2. Dry Air Density Variation with Altitude

Sea level pressure, p_0	101325 N/m ²
Sea level temperature, T_0	288.15 K
Hydrostatic constant	34.1631947 K/km

The standard sea level density that is calculated from the settings above is 1.225 kg/m³. The variation of dry air density and pressure - using the US standard atmosphere parameters - are shown in the Figure 1 and Figure 2 below.

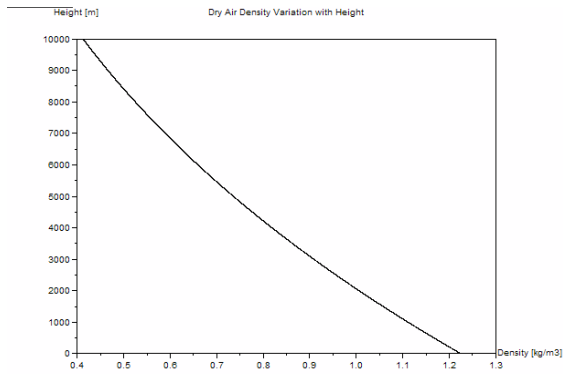


Figure 1: Dry Air Density Variation.

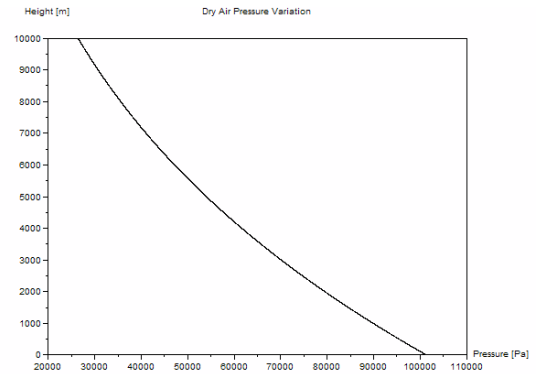


Figure 2: Dry Air Pressure Variation-

References

- [1] Murry L. Salby: *Fundamentals of Atmospheric Physics*, Academic Press, 1996, Elsevier Science
- [2] Ralph L. Carmichael, *The Hydrostatic Equations*, 2003 (internet note, www.pdas.com/coesa.htm)

3. Density of Moist Air

Even if moist air is less dense than dry air, the water vapor seldom represents more than a few percent of the air mass.

The model for moist air is based on the Dalton law for partial pressures, and is included in WindPRO in order to complete the description of the air density variation. The description is based on M. Salby [1], and is valid for air not condensed.

In WindPRO the user may input three different measures of the vapor:

- Relative humidity
- Dew point temperature
- Specific humidity

The user inputs are – however – always converted into a relative humidity – which is saved with the WindPRO project.

Density of Moist Air – Governing Equation

The density is determined as a mixture of dry air molecules and water vapour molecules

$$\rho_{air} = \frac{P_d(T)}{R_d T} + \frac{P_v(T)}{R_v T} \quad (1)$$

where ρ_{air} is the density [kg/m³]

P_d is the partial pressure of dry air [Pa]

P_v is the water vapour partial pressure [Pa]

R_d is the specific gas constant for dry air [J/(kgK)] = 287.05

R_v is the specific gas constant for water vapour [J/(kgK)] = 461.495

T is the temperature [K]

Saturation Vapor Pressure

In order to calculate the density of moist air, we need to know the saturated vapour pressure. This concept is related to the process of evaporation. Considering a closed container with water and air, the evaporation process will proceed until there are as many molecules returning to the liquid as there are escaping. When this balance is achieved then the vapour is said to be saturated (and the corresponding pressure denoted saturated vapour pressure). When the saturated vapour



Brokilde Wind Farm, Denmark.

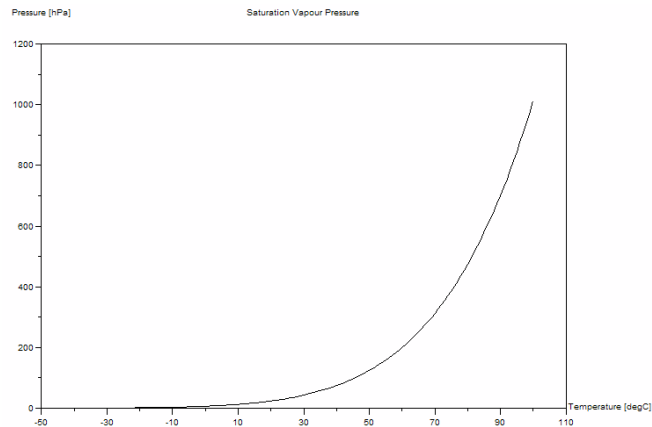


Figure 1: Variation of the saturation vapor pressure.

3. Density of Moist Air

pressure is equal to the atmospheric pressure, then the liquid is boiling.

In WindPRO, the saturated vapour pressure is calculated using an approximating polynomial, suggested by Herman Wobus. The polynomial was fitted from data from the Smithsonian Meteorological Tables by Roland List (6th edition), and is valid for temperature ranges from 50 °C to 100 °C. A graph of the saturated vapour pressure variation with temperature is found in Figure 1.

$$\begin{aligned}
 E_s(T) &= e_{s0} / p(T)^8 \\
 p(T) &= c_0 + T(c_1 + T(c_2 + T(c_3 + T(c_4 + T(c_5 + T(c_6 + T(c_7 + T(c_8 + T \cdot c_9)))))))) \\
 e_{s0} &= 6.1078 \\
 c_0 &= 0.99999683 \cdot 10^0 & c_1 &= -0.90826951 \cdot 10^{-2} & c_2 &= 0.78736169 \cdot 10^{-4} \\
 c_3 &= -0.61117958 \cdot 10^{-6} & c_4 &= 0.43884187 \cdot 10^{-8} & c_5 &= -0.29883885 \cdot 10^{-10} \\
 c_6 &= 0.21874425 \cdot 10^{-12} & c_7 &= -0.17892321 \cdot 10^{-14} & c_8 &= 0.11112018 \cdot 10^{-16} \\
 c_9 &= -0.30994571 \cdot 10^{-19}
 \end{aligned} \tag{2}$$

where T is the temperature in [°C]
 E_s is the saturated vapour pressure in [mb]

Different Measures of Humidity

The actual vapour pressure is now determined from either the dew point or the relative humidity. If the dew point, T_{dew} , is known, then the actual vapor pressure is simply

$$P_v(T) = E_s(T_{dew}) \tag{3}$$

If the relative humidity (RH) is known then the actual vapour pressure simply determined from the definition of the relative humidity. I.e. the relative humidity is simply the ratio of actual vapor pressure to the saturation vapor pressure at a given temperature.

$$P_v(T) = RH \cdot E_s(T) \tag{4}$$

The specific humidity is the relative concentration of vapor. It is defined from the following equation

$$SH = \rho_v / \rho = m_v / m \tag{5}$$

where ρ_v is the absolute concentration of vapor ($\rho_v = 1/v_v$)
 v_v is the specific volume of vapor ($v_v = V/m_v$, V is the volume and m_v is the mass of the vapor)
 ρ is the density of the air
 m_v and m is the masses of vapor and the mixture

In case that the specific humidity is known (SH) then the vapor pressure is calculated from (see M. Salby [1]):

$$\begin{aligned}
 r &= SH / (1 - SH) \quad \text{and} \\
 P_v &= (r / \varepsilon) \cdot P
 \end{aligned} \tag{6}$$

where r is the mixing ratio
 ε is the ratio of molar weights $\cong 0.622$

3. Density of Moist Air

Calculating the Air Density of Moist Air

The specific gas constant of the mixture of dry and moist air is expressed as:

$$\begin{aligned}
 R_{mix} &= (1 - SH) \cdot R_d + SH \cdot R_v \\
 &= (1 - SH) \cdot R_d + \frac{SH}{\epsilon} \cdot R_d \\
 &= \left(1 + \left(\frac{1}{\epsilon} - 1\right) SH\right) \cdot R_d
 \end{aligned} \tag{7}$$

Using this new gas constant with the equations (13) and (14) established in the previous Chapter 2, enables us to calculate the density of moist air. Note, it is an implicit assumption that the gas constant of the mixed gas remains constant through varying heights. This is presumably a reasonable assumption – because the correction due to moisture is very small.

Sample Calculation with US Standard Atmosphere Parameters

In the Figure 2 and Figure 3 the air density variation with height is shown for the U.S. Standard Atmosphere, 1976. Included are also different measures of relative humidity – which are shown to have only a quite small influence.

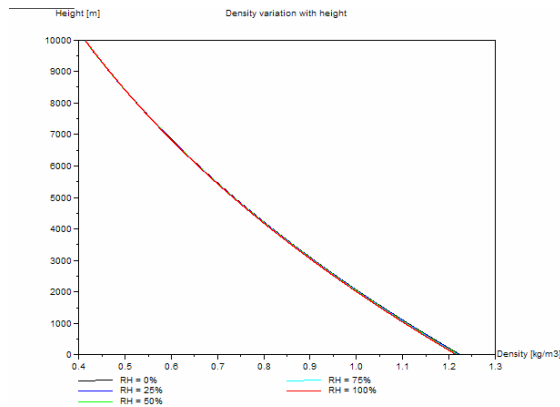


Figure 2: Moist Air Density Variation.

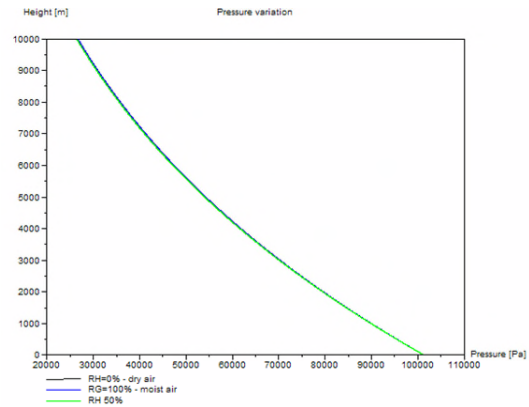


Figure 3: Moist Air Pressure Variation.

In Figure 4 the relative difference between dry air and moist air is shown for the first 5000 meters above the sea level. It is observed that the difference is below 0.7%. Again, the figure is calculated using the US Standard Atmosphere parameters.

3. Density of Moist Air

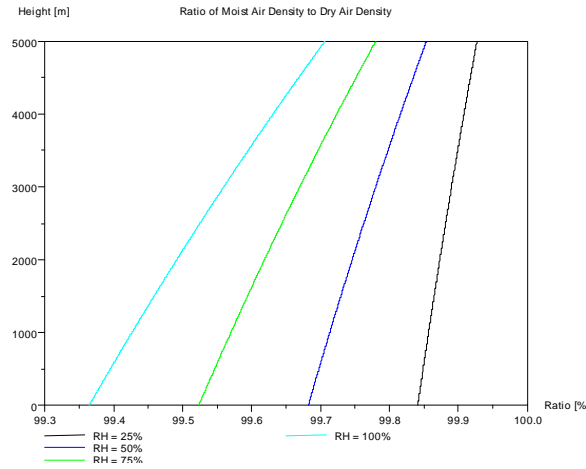


Figure 4: Ratio of Moist Air to Dry Air.

Air Density Variation with Humidity and Pressure (alternative equations)

The *CRC Handbook of Chemistry and Physics* [2] holds an alternative description of the of the moist air density variation with temperature, pressure and vapor pressure. The relation is:

$$D = 1.2929 \cdot (273.13 / T) [(B - 0.7383e) / 760] \quad (8)$$

where T is the absolute temperature (degK)
 B is the barometric pressure (mmHG)
 e is the vapor pressure (mmHG)

This equation (8) is now used to make a sensitivity study of the influence of moist air. This is done by calculating the ratio of moist air density to dry air density. This calculation is shown – for temperatures 5°C - 20 °C in the figures below. From the Figure 5 - Figure 8 it is observed, that the density influence of moist air is less than 1.7 percent for the considered temperature range. Higher temperatures yield a higher influence. For wind turbine applications the air moisture density influence could – in most cases – be ignored (i.e. it is a reasonable assumption to use the dry air properties).

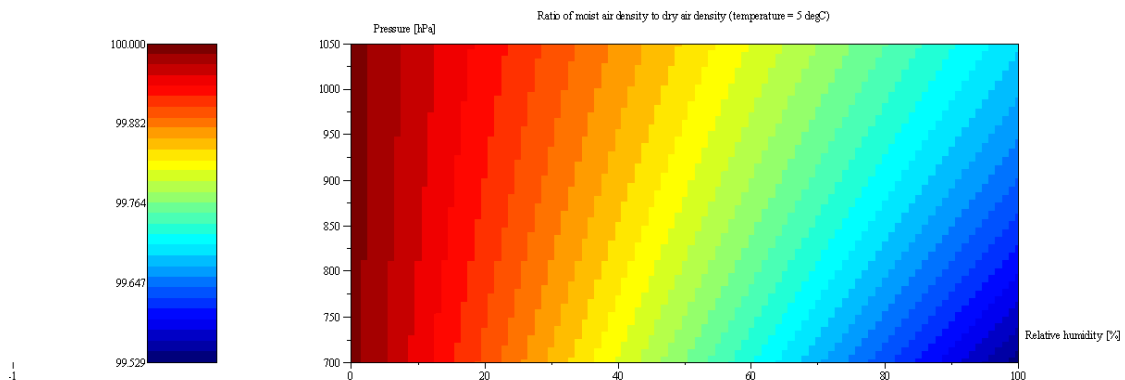


Figure 5: Density Ratio for Temperature = 5 deg C in [%].

3. Density of Moist Air

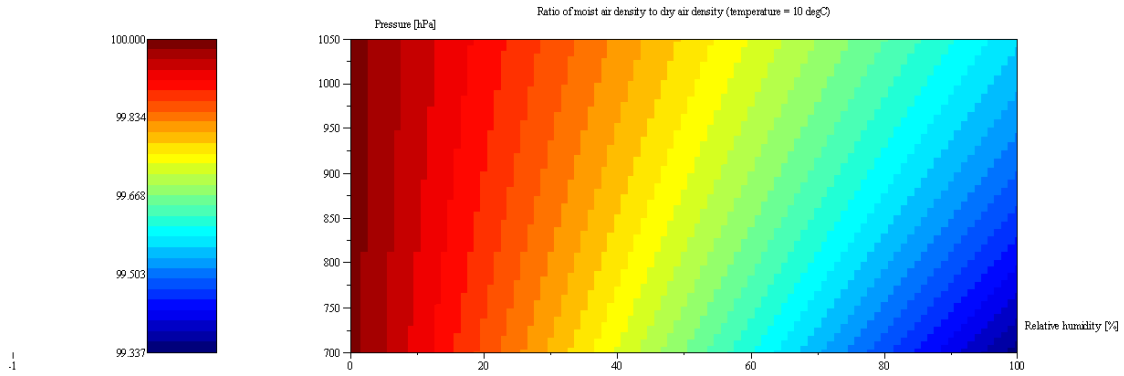


Figure 6: Density Ratio for Temperature = 10 deg C in [%].

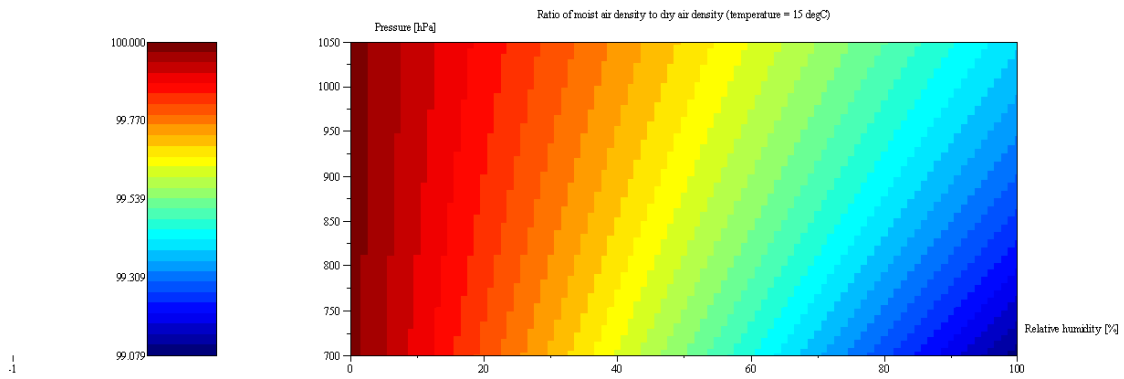


Figure 7: Density Ratio for Temperature = 15 deg C in [%].

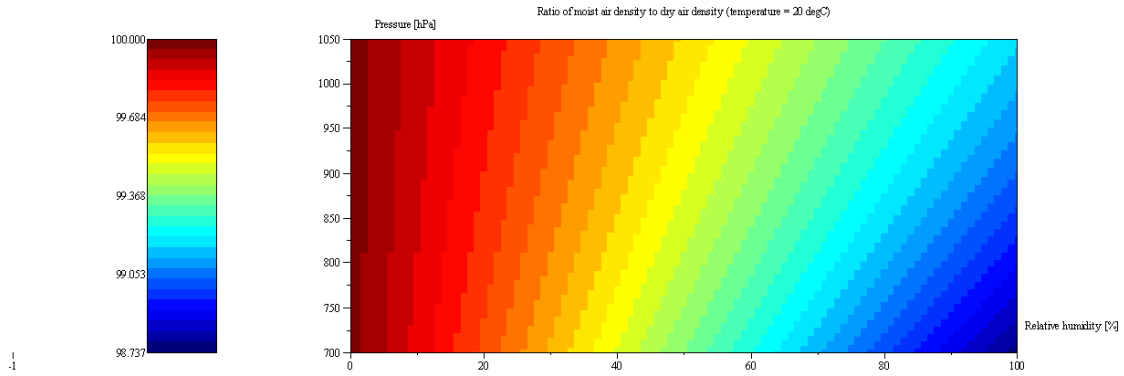


Figure 8: Density Ratio for Temperature = 20 deg C in [%].

References

- [1] Murry L. Salby: *Fundamentals of Atmospheric Physics*, Academic Press, 1996, Elsevier Science
- [2] *CRC Handbook of Chemistry and Physics*, CRC Press Inc., 60th edition, 1981,

4. User Guide

This chapter gives instructions on how to operate the air density calculator. Compared to earlier editions of WindPRO the air density calculation has with WindPRO 2.5 become rather more sophisticated and therefore also more complex for the user. The theoretical background is covered in the previous two chapters.

Default settings

WindPRO will by default try to calculate the air density of the site. If the user do nothing else but accept the suggestion from WindPRO the air density field will be greyed out with a hatching in the Auto / Individual box.

The default settings of the Air Density Calculator can be seen by pressing the Edit/View button and is shown below.

Input specifications

Calculation altitude (site) 394 meter ☒ From site and hub height

Annual mean temperature 3.3 °C At site altitude ☒ Auto from altitude/latitude

Other options

Load predefined data from Standard Atmosphere Climate database

Calculation result

Air density at calculation height 1.216 kg/m3

Use model to calculate individual WTG air density ☒

Advanced Mode >>

View graph >>

Ok Cancel

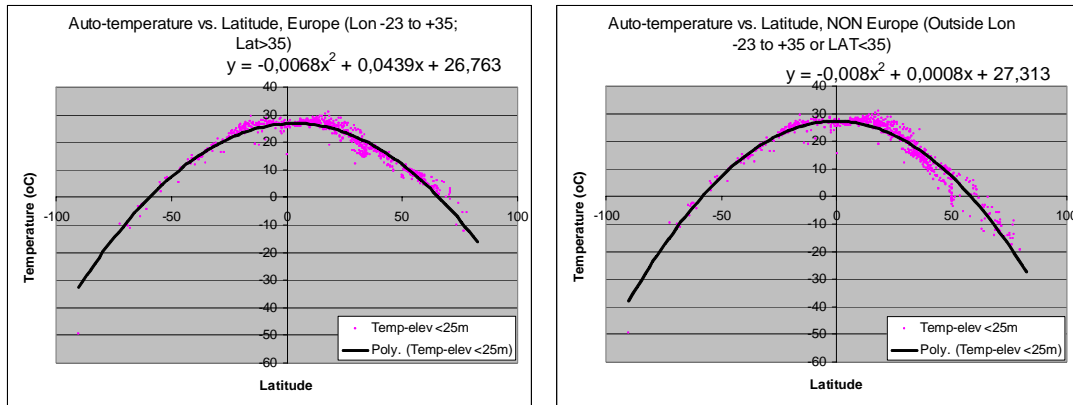
Since the individual WTG air density box is checked WindPRO will calculate an individual value for each of the turbines including the location of the key result.

The hatching of the “From site and hub height” box means that WindPRO calculates the height above sea level of the hubs of each of the turbines. The height in the greyed box is the height above sea level of the site data object + the key result height. Please note that WindPRO can only calculate the height above sea level for the turbines if the z coordinate is given in the WTG properties either through individual values or from the DHM. If a WAsP interface or an ATLAS calculation is made the heights for the turbines come from the site data object, for a METEO calculation they come from the Meteo object.

The hatching of the “Auto from altitude/latitude” means that WindPRO tries by itself to calculate a temperature at hub height. This is based on the latitude of the location and the height above sea level.

For the latitude WindPRO applies the location of the site to a best-fit relation between latitude and temperature at sea level. It is a sound assumption that temperature decreases with distance from the equator, but local climate factors can influence the temperature radically. The most prominent example is the difference between Europe and North America (and indeed Asia too) caused by the Gulf Stream. The temperature in Europe is for the same latitude significantly higher than in North America. For this reason a separate relation is used for Europe. The two relations are shown below.

4. User Guide



From this temperature at sea level the temperature at hub height is found using the temperature/height relationship discussed in the theoretical sections. The temperature shown in the greyed box is the temperature at key result height above the location of the site data object.

The air density in the calculation result box is the calculated air density at key result height above the site data object.

User control of the air density calculation

The user has a number of options for influencing the air density calculation. The options are discussed in the below case situation.

A fixed air density for the calculation.

If a fixed air density is wanted, to be used for all the turbines in the calculation you do not even have to access the air density calculator. Simply remove the hatching for “Auto/Individual” and type the wanted air density.

A fixed air density based on known temperature and height (the 2.4 way).

If you access the air density calculator, remove all the three visible hatchings and type in the temperature and height above sea level, then you get a calculated air density that will be used for all the turbines and the key result calculation. This is exactly what was done in earlier versions of WindPRO. This setup is shown below.

4. User Guide

Input specifications

Calculation altitude (site) meter ☐ From site and hub height

Annual mean temperature °C ☐ Auto from altitude/latitude

Other options

Load predefined data from

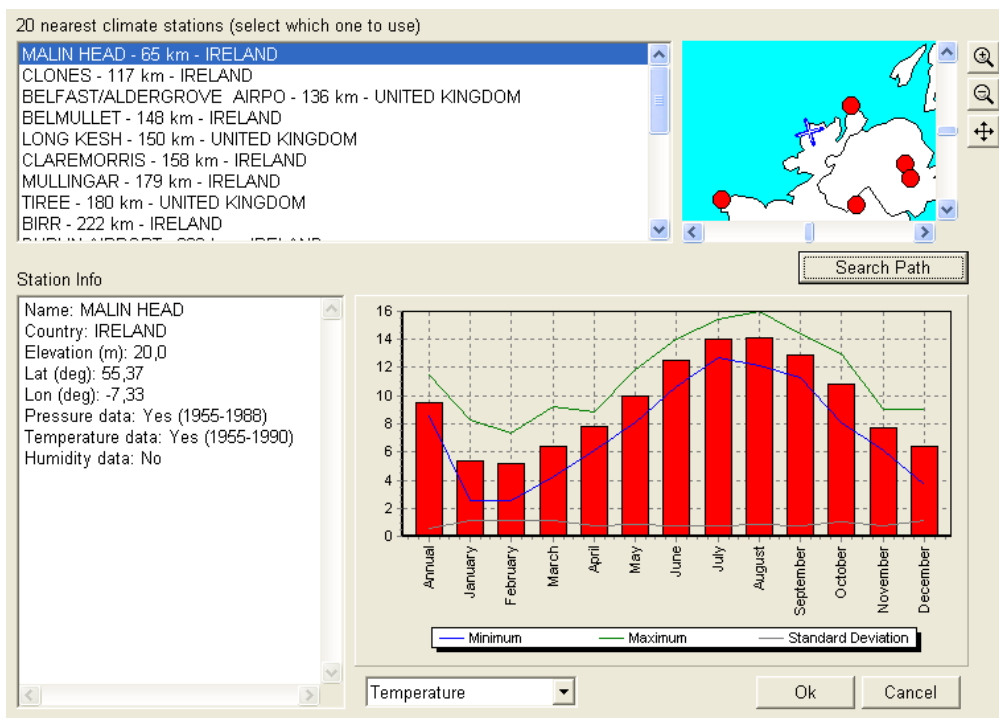
Calculation result

Air density at calculation height kg/m3

Use model to calculate individual WTG air density ☐

Using the climate data base

The user can access the climate database by pressing the “Climate database” button. This starts a search through the climate database for the 20 closest stations. When the search is done the following window appear.



The climate database is from the “Global Historical Climatology Network ” (vers. 1) from The Oak Ridge National Laboratory (Vose et.al. 1992). Highlighting a station either in the list or on the map lists the properties of this station in the lower two windows. Here profiles on temperature, pressure and humidity can be seen. Pressing Ok returns the average temperature and height above sea level to the air density calculator.

4. User Guide

The screenshot shows a software interface for calculating air density. It is divided into three main sections: 'Input specifications', 'Other options', and 'Calculation result'. In the 'Input specifications' section, 'Calculation altitude (site)' is set to 20 meters, and 'Annual mean temperature' is 9.5 °C. There are checkboxes for 'From site and hub height' and 'Auto from altitude/latitude', both of which are unchecked. The 'Other options' section has a 'Load predefined data from' dropdown set to 'Standard Atmosphere', with a 'Climate database' button next to it. The 'Calculation result' section shows 'Air density at calculation height' as 1.246 kg/m3. A checkbox 'Use model to calculate individual WTG air density' is checked. On the right side of the 'Calculation result' section, there are buttons for 'Advanced Mode >>', 'View graph >>', 'Ok', and 'Cancel'.

Input specifications	
Calculation altitude (site)	20 meter
Annual mean temperature	9,5 °C
<input type="checkbox"/> From site and hub height	
<input type="checkbox"/> Auto from altitude/latitude	
Other options	
Load predefined data from	Standard Atmosphere
Climate database	
Calculation result	
Air density at calculation height	1,246 kg/m3
<input checked="" type="checkbox"/> Use model to calculate individual WTG air density	
Advanced Mode >>	
View graph >>	
Ok	Cancel

Please note that the hatching is now removed from the height “from site and hub height” and temperature “auto from altitude/latitude” boxes. Instead the air density is calculated based on a specific height and temperature. The relations discussed in the theory section then translate this to an air density for each hub height and as shown as the calculation result for the key result height above the site data object.

For resetting the model press “Standard Atmosphere” and the settings returns to normal.

Using measurements on site.

If measurements were made on the site you can enter the average temperature measured and the height above sea level for the measurements (base height of the tower + the metering height). This requires that the two hatchings related to height and temperature is removed. Keeping the “Use model to calculate individual WTG air density” box hatched will then translate the air density to the hub height of each turbine.

This screenshot shows the same software interface as the previous one, but with different input values. In the 'Input specifications' section, 'Calculation altitude (site)' is now 395 meters, and 'Annual mean temperature' is 10.0 °C. The checkboxes for 'From site and hub height' and 'Auto from altitude/latitude' remain unchecked. The 'Other options' section is identical, with 'Standard Atmosphere' selected. The 'Calculation result' section now shows an 'Air density at calculation height' of 1.189 kg/m3. The 'Use model to calculate individual WTG air density' checkbox is still checked. The right-side buttons ('Advanced Mode >>', 'View graph >>', 'Ok', 'Cancel') are also present.

Input specifications	
Calculation altitude (site)	395 meter
Annual mean temperature	10,0 °C
<input type="checkbox"/> From site and hub height	
<input type="checkbox"/> Auto from altitude/latitude	
Other options	
Load predefined data from	Standard Atmosphere
Climate database	
Calculation result	
Air density at calculation height	1,189 kg/m3
<input checked="" type="checkbox"/> Use model to calculate individual WTG air density	
Advanced Mode >>	
View graph >>	
Ok	Cancel

Advanced mode

Pressing the “Advanced Mode” button gives you the full view of the air density calculator. Here you have the full flexibility to design your own air density calculator.

4. User Guide

Input specifications

Calculation altitude (site) meter ☐ From site and hub height

Annual mean temperature °C ☐ Auto from altitude/latitude

Specification altitude above sea level (base) meter

Temperature lapse rate (gradient) °C per 100 m ☒ Use default = 0.65

Air pressure at base height hPa ☒ Use standard model

Humidity Relative humidity [%] ☒ Use default = 0.00

Other options

Load predefined data from

Calculation result

Air density at calculation height kg/m³

Use model to calculate individual WTG air density ☒

The user can select either a temperature measured at the height where the calculation should be used or at a base level where the temperature is known. This could be the case where temperature is known from a station at 20 m above sea level ("Specification altitude above sea level (base)"), but should be used in either hub height (hatch the "Use model to calculate individual WTG air density" box) or a specific height to be used for all the turbines (un-hatch the "Use model to calculate individual WTG air density" box and type a height for "Calculation altitude". Note the difference between calculating at base altitude and site altitude. It takes the height from either of the two boxes.

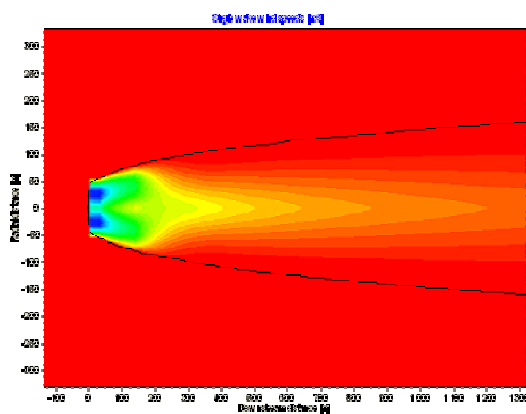
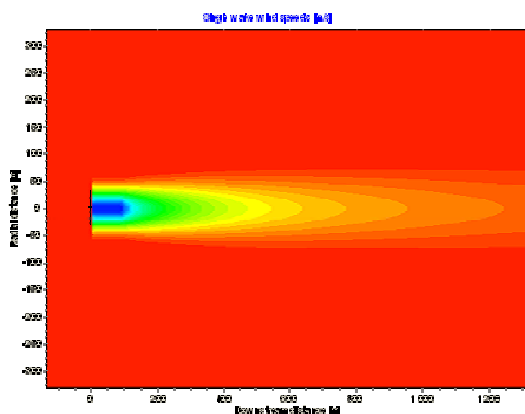
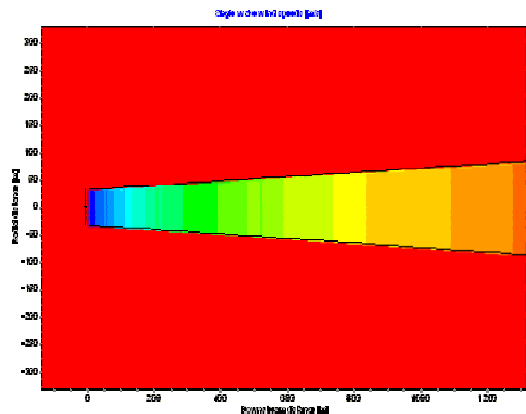
The temperature lapse rate, the air pressure at base height and the humidity can by removing the appropriate hatching be adjusted. EMD suggests using the default values unless more precise site-specific values are obtained. The theoretical background for these figures can be found in the theory section. View graph show the temperature-height relationship with current settings.

References

Vose et.al.: 1992, "The Global Historical Climatology Network", The Oak Ridge National Laboratory, prepared for U.S. Department of Energy.

WindPRO / PARK

Introduction to Wind Turbine Wake Modelling and Wake Generated Turbulence



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Front cover

The front cover shows a wake development behind a single turbine. The wake velocities are calculated using the N.O. Jensen PARK model, the Ainslie model (eddy viscosity) and the G.C. Larsen model.

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1. Introduction to Wake Modelling

This paper gives a brief introduction to the concept of wake modelling for arrays of wind turbines. While WindPRO offers the opportunity to calculate the wind farm wakes with a number of different wake models, the user must choose and configure the model that fits the best into the area of application. This report gives selected background theory of single and multiple wake modelling and also on the implementation of the specific wake models. Descriptions of the different wake models are found in the succeeding chapters.



Figure 1: A cluster of four >2 MW semi-offshore turbines in Frederikshavn, Denmark.

Introduction

When the turbine extracts power from the wind, a wake evolves downstream of the turbine. If another nearby turbine is operating within this wake, the power output for this downstream turbine is reduced when comparing to the turbine operating in the free wind. This reduction of power output is – on an annual basis – typically in the range of approximately 2% - 20%, depended on the wind distribution, the wind turbine characteristics and the wind farm (array) geometry.

The turbines operating in the wake are not only subjected to a decreased wind speed but also increased dynamic loading – arising from the increased turbulence induced by the upstream turbines. This increased turbulence must be accounted, when selecting a turbine suitable class of turbines. This is typically done though the specifications in the international codes – e.g. the IEC-61400-1 code for wind turbine structures.

The models available in WindPRO are currently all single wake models, i.e. models capable describing the flow downstream of one turbine. When having multiple turbines, the results from the single wake models are aggregated into a combined result by using empirical combination rules.

The Wake after an Idealized Turbine

Assuming an idealized turbine – where flow around and behind the turbine is without rotation and friction - it is possible to derive some general and important equations describing the wake wind speeds. For further details please consult the publication by Andersen et al. [1]. The derivation is based on the simplified Bernoulli equation, stating that the mechanical energy per unit mass – along a streamline - is conserved:

$$\frac{\rho V^2}{2} + p = H \quad (1)$$

where ρ is the air density
 V is the wind speed
 p is the pressure
 H is the total energy (constant along any streamline)

1. Introduction to Wake Modelling

The Bernoulli equation gives the relation between pressure and wind speed, as the total pressure is constant along a streamline (streamline = a line which is drawn, such as it is always tangent to the velocity vector). Using the Bernoulli equation just before and after the rotor gives us two equations:

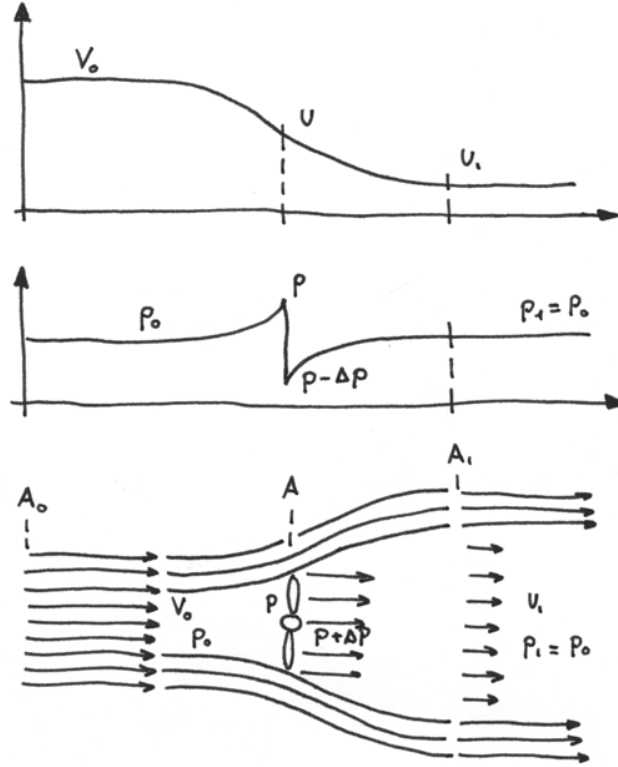


Figure 2: Flow near an idealized turbine: velocity and pressure.

$$\begin{aligned} p_0 + \frac{1}{2}\rho V_0^2 &= p + \frac{1}{2}\rho u^2 \quad \text{and} \\ p - \Delta p + \frac{1}{2}\rho u^2 &= p_0 + \frac{1}{2}\rho u_1^2 \end{aligned} \quad (2)$$

These two equations are then subtracted to yield the drop in pressure over the rotor plane

$$\Delta p = \frac{1}{2}\rho(V_0^2 - u_1^2) \quad (3)$$

Another method for calculating the drop in pressure, Δp , is expressing the drop as the change in momentum of the mass of air passing through one square meter of the rotor area per second (actually by considering the second law of Newton). This yield

$$\Delta p = \rho u(V_0 - u_1) \quad (4)$$

Now equating the equations (3) and (4) gives an expression for the wind speed in the rotor plane:

$$u = \frac{1}{2}(V_0 + u_1) \quad (5)$$

i.e. the velocity in the rotor plane is exactly the average of the far upstream and the far downstream wind speed.

1. Introduction to Wake Modelling

The axial thrust force – i.e. the force acting in the direction of the wind – which is denoted, T , is calculated from knowledge of the pressure difference:

$$T = \Delta p \cdot A \quad (6)$$

where T is the thrust force
 Δp is the difference force
 A is the rotor area

Now defining a ‘axial interference factor’ – a , which is

$$u = (1 - a) V_0 \quad \text{and thus} \quad u_1 = (1 - 2a) V_0 \quad (7)$$

Inserting the equation (7) and equation (3) into the thrust definition equation (6) yields

$$T = 2\rho a(1 - a)V_0^2 A \quad (8)$$

Now defining a thrust coefficient, $C_T = 4a(1 - a) \Rightarrow a = 1/2 \pm 1/2 \sqrt{1 - C_T}$, gives

$$T = (1/2) \cdot \rho \cdot V_0^2 A \cdot C_T \quad (9)$$

Inserting the expression for a (and C_T) into the equation (7), this yield

$$u_1/V_0 = (1 - 2a) = \sqrt{1 - C_T} \quad (10)$$

With the equation (10) we now have a relation established between downstream wake wind velocity – u_1 , the turbine thrust coefficient – C_T – and the free wind speed – V_0 . This relation is - using an assumption of the downstream wake expansion – used for making simple and computationally very efficient turbine wake models – like the N.O. Jensen PARK model.

Wake Expansion: When applying the continuity equation in relation with the equation (6) and (10) which are expressions for the wind speed in the rotor plane (u) and far downstream of the turbine (u_1), then an expression for the so-called expanded diameter can be derived [2]:

$$D_{\text{exp}} = RD\sqrt{(1 - a)/(1 - 2a)} \quad (11)$$

Turbulent mixing makes the wind speeds recover to the free wind speeds at some downstream distance, but the equations (10) and (11) can be used to gain insight in the wake expansion rate.

Important Wake Model Parameters

The wake models require different internal wake model parameters as input - as well as a varying number of additional parameters describing the terrain and/or wind climate conditions. Input parameters to a wake model can be turbulence intensity and roughness length. Typically, one would assume that such parameters are depended on the roughness class (or roughness length). In the lack of the preferred measured data, the table below suggests corresponding estimated wake model parameters.

1. Introduction to Wake Modelling

Terrain classification	Roughness Class	Roughness Length	Wake Decay Constant	Ambient Turbulence at 50 m* $A_x = 1.8$	Ambient Turbulence at 50 m** $A_x = 2.5$	Additional detailed description
Offshore. Water areas	0.0	0.0002	0.040	0.06	0.08	Water areas, oceans and large lakes. General water bodies.
Mixed water and land	0.5	0.0024	0.052	0.07	0.10	Mixed water and land. Also applies to the very smooth terrain
Very open farmland	1.0	0.0300	0.063	0.10	0.13	No crossing hedges. Scattered buildings. Smooth hills.
Open farmland	1.5	0.0550	0.075	0.11	0.15	Some buildings. Crossing hedges with 8 m height with distance 1250 m apart.
Mixed farmland.	2.0	0.1000	0.083	0.12	0.16	Some buildings. Crossing hedges 8 m high with distance 800 m apart.
Trees and farmland	2.5	0.2000	0.092	0.13	0.18	Closed appearance. Dense vegetation. 8 m hedges 250 m apart.
Forests and villages	3.0	0.4000	0.100	0.15	0.21	Villages, small towns and much closed farmland. Many high hedges. Forests.
Large towns and cities	3.5	0.8000	0.108	0.17	0.24	Large towns, cities with extended build up areas.
Large build up cities	4.0	1.6000	0.117	0.21	0.29	Large cities with build up areas and high buildings.

* The turbulence intensity is actually calculated based on the assumption of homogeneous terrain with a surface roughness equal to the roughness length. Input to the calculation is also the turbulence measurement height – see the equation below (here based on $A_x = 1.8$, see the equation below).

** Calculated using $A_x = 2.5$, see the equation below.

WindPRO 2.5 assumes that $A_x = 2.5$. Please note that if - during the automated conversions in WindPRO - a terrain classification is exceeding the limits in the table (either the ‘Offshore’ or the ‘Large build up cities’) then the nearest tabular value is chosen.

Estimating the Turbulence Intensity

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain - be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [3, section 3.1.2]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[\frac{1}{\ln[z / z_0]} \right]$$

The value of A is reported to vary approximately between 2.5 to 1.8. κ is the Von Karman constant, which is equal to 0.4. In DS 472 the product between A_x and κ is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC, the characteristic data needed is actually a mean value plus one standard deviation.

Wake Model Limitations – Large Wind Farms and Thrust Coefficient

The wake models are calibrated and tested in small to medium sized wind farms – i.e. wind farms with up to approximately 50-75 turbines. For very large wind farms – 75 turbines or more – the turbines may

1. Introduction to Wake Modelling

influence the surrounding upper air wind climate (Geostrophic winds). In that case, special modelling should be applied – e.g. by ‘artificially’ increasing the roughness within the wind farm.

An important parameter for most models is the thrust coefficient, C_t . The thrust coefficient is used to relate the free wind speed to the downstream wake wind speed through the equation $u_1/V_0=(1-C_t)^{0.5}$. Since the square root is taken, it might be a requirement – depended on the wake model - that the value of C_t is less than 1.0.

Calculating the Annual Energy Production (AEP)

Below is a short description of the algorithm used for calculating the AEP. The wake calculations and the annual energy production (AEP) calculation are actually integrated within the same calculation loop. It is calculated according to the algorithm as outlined in the pseudo code below. Note, that the algorithm assumes that the free wind distribution is based on a modeled Weibull distribution. If measured data is used instead, then the joint distribution table data is used.

```
for iTurbine = 1: N

    Select i Turbine

    Wake Combination for iTurbine calculated (looking at all upstream turbines) /
    Inflow conditions calculated (turbulence and velocity deficits) /
    looping turbines 1 :( iTurbine-1)

    save iTurbine_ThisLoopWakeWSP

    Wake Model for iTurbine:
    - Velocity deficit caused by iTurbine on all downstream turbines
      calculated and stored (from iTurbine+1 to N)

    Incremental AEP calculation (assuming here 12 wind-sectors):
    Find Sector from SectorAngle
    Lookup in weibull table: BinProb=frequency(sector)*AngleStep/30*
    ( F(ThisLoop_WSP)-F>LastLoop_WSP) );
    // F is the cummulative weibull distribution
    AvgPowerInBin:=(Power(iTurbine_ThisLoopWakeWSP)-
    Power(iTurbine_LastLoopWSP)) / 2;
    AEP_iTurbine:=AEP_iTurbine+AvgPowerInBin*8760*IncProb;

    save iTurbine_LastLoopWakeWSP=iTurbine_ThisLoopWakeWSP

end for
```

References

- [1] P.S. Andersen, U. Krabbe, P. Lundsager, H. Petersen, *Basismateriale for Beregning af Propelvindmøller*, Risø-M-2153(rev.), Forsøgsanlæg Risø, Januar 1980.
- [2] J.G. Schepers: ENDOW: *Validation and improvement of ECN's wakemodel*, ECN-C-03-037, March 2003.
- [3] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition, Risø National Laboratory

2. Introduction to the N.O. Jensen Wake Model

The N.O. Jensen wake model is a simple single wake model. The model is documented in the paper ‘A Simple Model for Cluster Efficiency’ by I. Katić et al [1] and is based on the assumption of a linearly expanding wake diameter. This note gives an introduction to the N.O. Jensen wake model and how it is implemented in WindPRO.

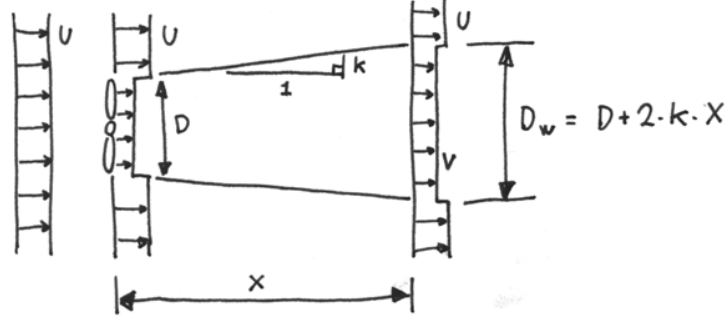


Figure 1: The N.O. Jensen wake model - overview.

Single Wake Calculation

When calculating the velocity deficit, the reduced wind speed, V , downwind of the turbine is derived from:

$$1 - V/U = \left(1 - \sqrt{1 - C_T}\right) / \left(1 + 2kx/D\right)^2 \quad (1)$$

Comparing this equation with the previously derived equation for the wind speed just downwind of the turbine (2), it is obvious that the assumption is a linearly expanding wake width.

$$V_0/U = \sqrt{1 - C_T} \quad (2)$$

where

V_0 is the wind speed directly after the turbine of consideration

However it is noticed, that it is not the actual wake wind velocity that is subject for this expansion assumption, but rather the velocity deficit $\delta V_i = (1 - V_i/U)$. Note, that the velocity deficit is defined through the free wind speed, U .

A plot from a calculation in WindPRO is shown in Figure 2. Note, that the wake velocity deficit is uniform given a certain downstream position.

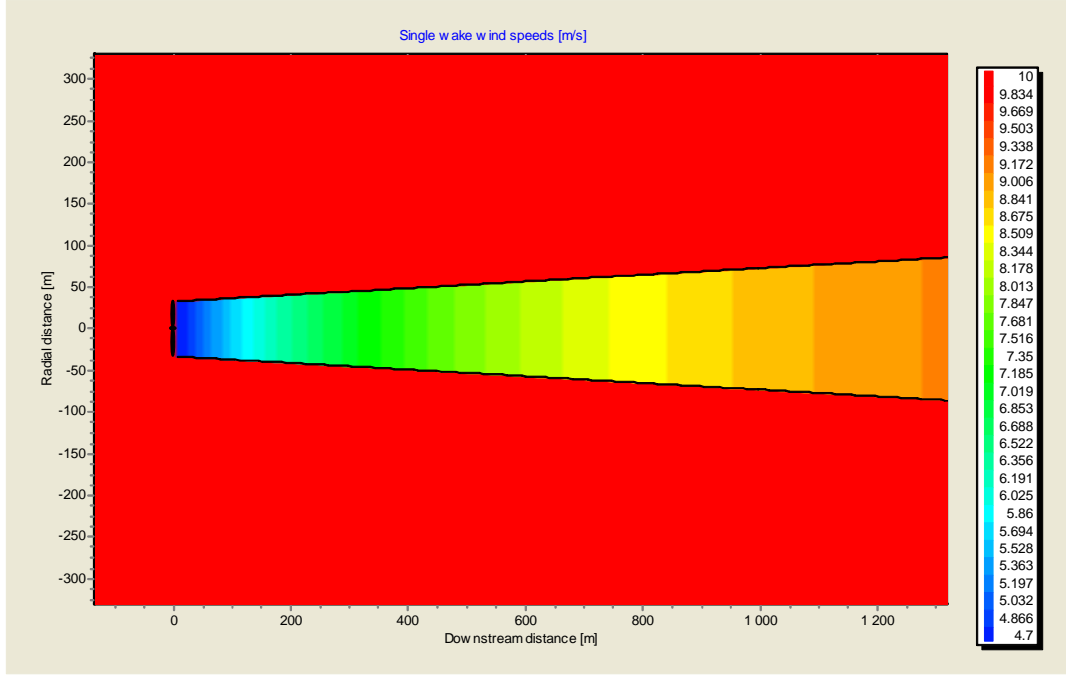


Figure 2: N.O. Jensen wake model - wake development after a single turbine.

Multiple Wake Calculation

Katic et al. [1] suggests, that multiple wakes are calculated through the ‘sum of squares of velocity deficits’ wake combination model. Thus, the N.O. Jensen model initially implemented in the WindPRO PARK module as well as the WAsP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution. The combined effects of multiple wakes are found as:

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2}$$

This model is treated in a succeeding chapter.

References

[1]I. Katić, J. Højstrup & N.O. Jensen, *A Simple Model for Cluster Efficiency*, European Wind Energy Association, Conference and Exhibition, 7-9 October 1986, Rome, Italy.

3. Introduction to the Ainslie Wake Model (Eddy Viscosity Model)

Introduction

The wind turbine wake application of an axi-symmetric formulation of the time averaged Navier Stokes equations with an eddy viscosity closure was initially made by Ainsley [3]. The application uses cylindrical coordinates and an assumption of incompressible fluid. A graphical overview of the model setup is shown in Figure 1.

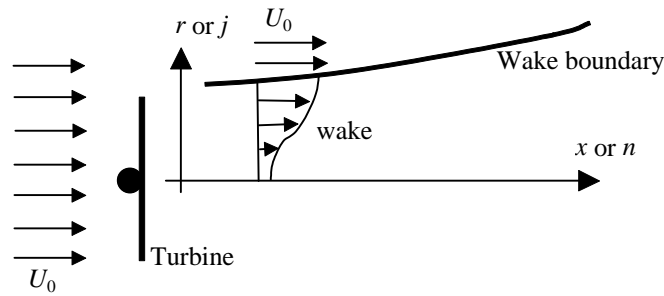


Figure 1: Flow around the turbine.

A result from an implementation of the model is shown in Figure 2, where the wake development behind a turbine with a 50 meters rotor is shown. The calculation is initiated at distance two-rotor diameters downstream (100 meters). Note, that the model calculates the flow through half of the rotor as indicated on Figure 1); this is due to the symmetry assumption used within the model. The free stream velocity is 8.0 m/s (as shown in the legend to the left), while the minimum velocity behind the turbine is 6.5 m/s. The trust coefficient – for this sample calculation – was set to 0.7.

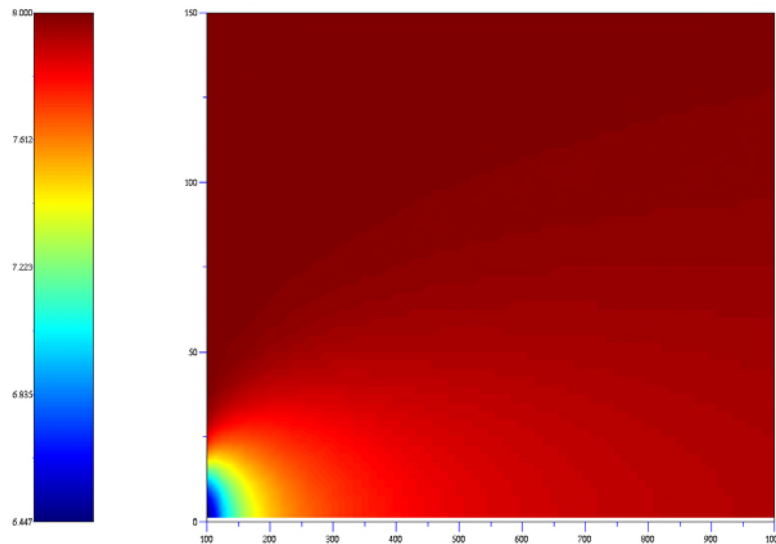


Figure 2: Wake development behind a turbine with a 50-meter rotor. Note that the calculation is initiated at 2·RD downstream.

Nomenclature

I_T	turbulence intensity
v	mean (averaged) velocity in radial direction
u	mean (averaged) velocity in axial direction
U_0	mean (averaged) velocity in free flow

σ_u	standard deviation of wind speed process
\overline{uv}	Reynolds stress
$\epsilon(x)$	eddy viscosity

Continuity Equation (axisymmetric case: $\partial/\partial\phi=0$)

The continuity equation in cylindrical coordinates is (Shames [1]):

$$\frac{1}{r} \frac{\partial rv}{\partial r} + \frac{\partial u}{\partial x} = 0$$

Navier Stokes Equation

In the thin layer approximation and using cylindrical coordinates, the Navier Stokes equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{1}{r} \frac{\partial(\overline{ruv})}{\partial r}$$

The last part of the equation above refers to the change in acceleration and thereby momentum. It is not possible to describe this contribution using the velocities in the averaged flow. The part is due to the change in momentum caused by the turbulent fluctuations.

Eddy viscosity (or turbulent exchange coefficient for momentum)

The eddy viscosity is defined in Tennekes and Lumley [2], and is used for establishing an interaction between mean flow and turbulent eddies.

$$-\overline{uv} = \epsilon(x) \frac{\partial u}{\partial r}$$

According to Ainslie [3], the eddy viscosity, $\epsilon(x)$, is adequately described by a length scale $L(x)$ and a velocity scale $U(x)$.

$$\epsilon(x) = L(x) \cdot U(x) + \epsilon_a$$

The length and velocity scales are taken to be proportional to the wake width b and the velocity difference across the wake shear layer (i.e. independent of r). ϵ_a is the contribution from ambient turbulence to the eddy viscosity. The length scales are determined by:

$$L(x) \cdot U(x) = k_1 \cdot b \cdot (U_0 - u_0(x))$$

3 case studies showed $k_1 = 0.015$.

Boundary conditions

Ainsley [3] gives the boundary conditions at two rotor diameters downstream of the turbine. The BC at this section is given as a Gaussian velocity profile with the input of initial velocity deficit D_M and wake width b :

$$1 - \frac{U}{U_0} = D_M \exp\left[-3.56 \cdot \left(\frac{r}{b}\right)^2\right]$$

Empirical data (wind tunnel studies) showed the following equations may be used for determining the velocity deficit and the wake width (A is ambient turbulence intensity in percent):

$$D_M = C_T - 0.05 - (16C_T - 0.5)A / 1000$$

$$b = \frac{3.56C_T}{8D_M(1 - 0.5)D_M}$$

Other authors specify a boundary condition where the initiation position (downwind position) varies. In Lange et al [4] reference to a study made by Vermeulen [5] is made. Vermeulen suggests that the near wake length is modeled through contributions from ambient turbulence, rotor generated turbulence and shear generated turbulence. The near wake length is divided into two regions; the first x_h is modeled as:

$$x_h = r_0 \left[\left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_\lambda^2 + \left(\frac{dr}{dx} \right)_m^2 \right]^{-0.5}$$

where r_0 is an 'effective' radius of an expanded rotor disc, $r_0 = [D / 2] \sqrt{(m+1) / 2}$ and $m = 1 / \sqrt{1 - C_t}$
 D is the rotor diameter
 C_t the thrust coefficient

The different contributions in the equation above are calculated as:

$$\left(\frac{dr}{dx} \right)_a^2 = \begin{cases} 2.5I + 0.05 & \text{for } I \geq 0.02 \\ 5I & \text{for } I < 0.02 \end{cases} \quad \text{ambient turbulence}$$

$$\left(\frac{dr}{dx} \right)_\lambda^2 = 0.012B\lambda \quad \text{rotor generated turbulence}$$

$$\left(\frac{dr}{dx} \right)_m^2 = [(1-m)\sqrt{1.49+m}] / (9.76(1+m)) \quad \text{shear - generated turbulence}$$

where I is the ambient turbulence intensity
 B is the number of rotor blades
 λ is the tip speed ratio

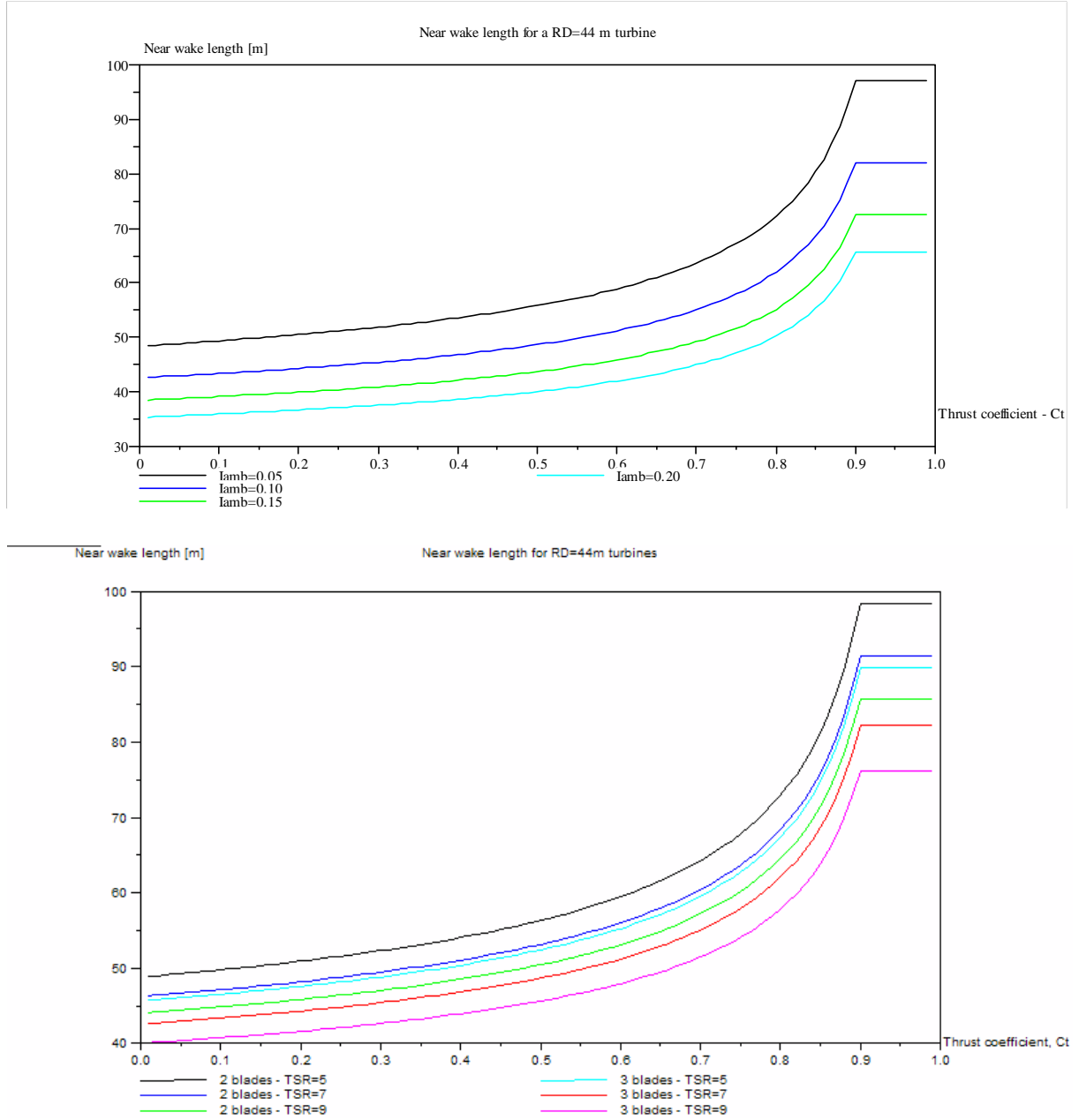


Figure 3: Near Wake Length for a 44 meter Rotor-Diameter Turbine using the Vermeulen Equations. Top: Sensitivity to ambient turbulence. Bottom: Sensitivity to type of turbine and tip speed ratio. Note, that the near wake length is decreasing with increasing ambient turbulence levels.

When the first near wake region, x_h , have been calculated, one can calculate the full near wake length, x_n , by:

$$x_n = \frac{\sqrt{0.212 + 0.145m}}{1 - \sqrt{0.212 + 0.145m}} \frac{1 - \sqrt{0.134 + 0.124m}}{\sqrt{0.134 + 0.124m}} x_h$$

Lange [4] reports that the equations save a singularity at about $C_t=0.97$, so it is suggested that for C_t 's larger than 0.9, then the value for C_t equal to 0.9 is used. A sample calculation for a 44-meter rotor diameter turbine is shown in

Numerical solution method

The differential equation is solved using a finite difference method using a generalized Crank-Nicholson scheme. The solution procedure followed is outlined in Wendt [6]. The numerical solution method used for solving the Navier Stokes equation is made by replaced the differential equation with the finite difference approximations. This approximation introduces truncation errors into the equation.

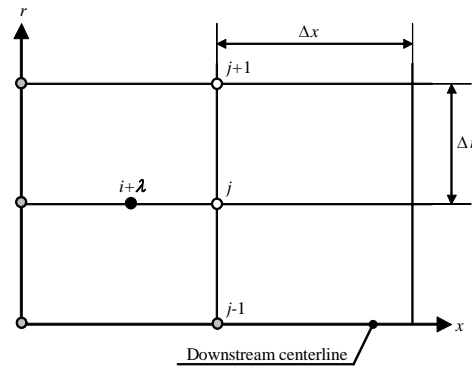


Figure 4: Grid for the generalized implicit method.

Outline of the Solution Procedure

The solution of the partial differential equations invokes an iterative solution procedure. From the boundary condition, the continuity equation is solved. Then the downstream momentum equation is solved in order to get the next downstream velocities. This solution is obtained through an iterative process – the iteration is stopped when convergences is achieved.

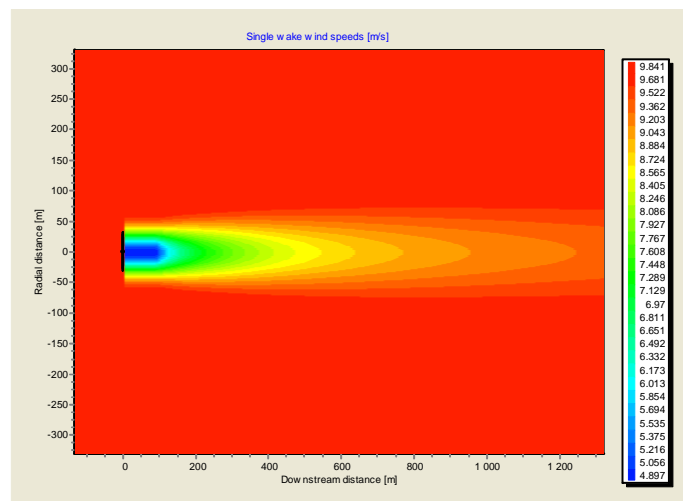


Figure 5: Eddy viscosity model – calculation from WindPRO.

A plot from WindPRO is shown in Figure 5. Note, that wind speeds within the near wake zone are approximated through the solution at the near wake distance.

References

- [1] Shames, Irving H.: *Mechanics of Fluids*, McGraw-Hill International Editions, 1992
- [2] Tennekes, H. & J.L. Lumley, *A First Course in Turbulence*, The MIT Press, 1972

- [3] Ainslie, J.F: *Calculating the flowfield in the wake of wind turbines*, Journal of Wind Engineering and Industrial Aerodynamics, 27 (1988), 213-224.
- [4] Lange, Bernard; H.P. Waldl; A.G. Guerrero; D. Heinemann & R.J. Barthelmie: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003 6:87-104.
- [5] Vermeulen, P.E.J: *An experimental analysis of wind turbine wakes*, Proceedings of the International Symposium on Wind Energy Systems, Lyngby, 1980.
- [6] Wendt, John F. (Ed.): *Computational Fluid Dynamics – An introduction*, 2nd Edition, Springer-Verlag, 1996.

4. Introduction to the G.C. Larsen Model (EWTS II)

Introduction

This model is a semi analytical model – derived from asymptotic expressions from Prandtl’s rotational symmetric turbulent boundary layer equations. Because of the asymptotic expressions, the model might be somewhat conservative for close spacings. The model is reported in [1] and is also the recommended wake model – for use with wake loading - in the project report from the European Wind Turbine Standards II Project, an EU-funded project finalized in 1999 [2]. This introduction is based on the EWTS-report [2]. A online introduction of the model can be found in the Risø report ‘A simple wake model’ [3] – this report is available online.

Model Equations

Assuming that similarity exist between deficits at different downstream positions and only moderate velocity deficits, then the wake radius can be described by:

$$R_w = \left[\frac{35}{2\pi} \right]^{1/5} [3c_1^2]^{1/5} [C_T Ax]^{1/3} \quad (1)$$

where

c_1 is a non-dimensional mixing length, described by $c_1 = l(C_T Ax)^{-1/3}$
 l is Prandtl’s mixing length

The c_1 parameter does – according to reference [2] – to some degree separate the rotor drag dependence and thus the c_1 is expected to be relative insensitive to the design and size of the rotor. An alternative and approximated specification of the c_1 parameter is found in section 5.1 of the EWTS II report [2], where the parameter is estimated as seen below in equation (2). This specification is adopted in WindPRO.

$$c_1 = \left[\frac{D}{2} \right]^{-1/2} (C_T Ax_0)^{-5/6} \quad (2)$$

where

C_T is the thrust coefficient
 A is the rotor area
 D is the diameter of the upstream rotor
 x_0 is an approximation parameter, determined by the equation (3) below

$$x_0 = 9.5D / \left(\left(\frac{2R_{9.5}}{D} \right)^3 - 1 \right) \quad (3)$$

In the equation above the $R_{9.5}$ parameter is determined as:

$$\begin{aligned} R_{9.5} &= 0.5[R_{nb} + \min(h, R_{nb})] \\ R_{nb} &= \max(1.08D, 1.08D + 21.7D(I_a - 0.05)) \end{aligned} \quad (4)$$

where

I_a is the ambient turbulence intensity at hub height

The wake boundary condition is satisfied, so that the wake radius at the rotor position equals the rotor diameter. Furthermore, empirical boundary condition is applied at 9.5 rotor diameters downstream, where the wake radius is determined from the equation (4). The equation ensures that the minimum turbulence intensity equal to 5% is used, and it essentially states that the wake expansion is dominated by ambient turbulence. The blocking effect of the ground is taken into account by using the design wake radius $R_{9.5}$ in eq. (4) – including the mean of R_{nb} and the minimum of the hub height and R_{nb} .

Mean Wind Velocity Deficit: The mean wind deficit is determined from the expression (5),

$$\Delta V = -\frac{V_a}{9} \left(C_T A x^{-2} \right)^{1/3} \left\{ r^{3/2} \left(3c_1^2 C_T A x \right)^{-1/2} - \left(\frac{35}{2\pi} \right)^{3/10} \left(3c_1^2 \right)^{-1/5} \right\}^2 \quad (5)$$

where

V_a is the ambient mean wind velocity at hub height

Modified Near Wake Description

The G.C.Larsen wake model includes the option of having a semi-empirical near wake description (second order approach) – enabling the user to model the near wake with a ‘double peak’ velocity profile. This approach is described in detailed in reference [1]. Using the second order option may give a more precise near wake description, especially for densely space turbines. The far wake is not modified.

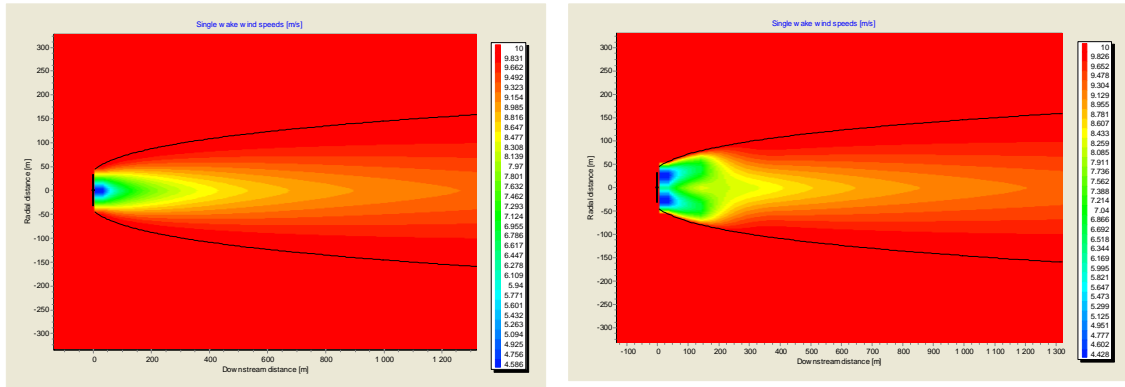


Figure 1: G.C.Larsen wake model (left: First order approach, right: second order approach)

References

- [1] G.C.Larsen, J. Højstrup, H.A. Madsen, *Wind Fields in Wakes*, EUWEC '96, Gothenburg, 1996.
- [2] *European Wind Turbine Standards II*, ECN-C-99-073, 1999
- [3] Larsen, G.C, *A simple wake calculation procedure*, RISØ-M—2760, Risø National Lab., Roskilde (Denmark), (online: <http://www.risoe.dk/rispubl/VEA/veapdf/ris-m-2760.pdf>)

5. Wake Combination Models

Today (2005), most wake models are still single wake models. Thus, in order to obtain a usable result for wind farms with many turbines, these single wakes must be combined into a combined effect. This is done by purely empirical means, using different wake combination models.



Figure 1: Horns Rev offshore wind farm.

Introduction

Two problems occur when trying to combine the results from a several single wake model into one single downwind wind speed:

1. Since the results from many of the single wake models are non-uniform distributed velocities or velocity deficits, these results must be averaged or combined into an efficient (uniform) wind speed. This is necessary, because the wind turbine power output is to be estimated from the available power curves.
2. When the downwind velocities are determined through one single wake calculation for each turbine, the single wake results must be added into a combined effect.

Ad. 1: Averaging of the Single Wake results

The output from many wake calculations is a non-uniform velocity field. However in order to calculate the power output from a measured power curve, the velocity field must be averaged over the rotor area. In WindPRO, a squared momentum deficit approach is used to calculate this reduction. This approach is similar to the one reported by Lange et. al [1].

$$(u_0 - u_{rotor})^2 = \frac{1}{A} \int_{rotor} (u_0 - u_w)^2 dA \quad (1)$$

where u_0 is the free stream velocity
 u_{rotor} is the averaged velocity at the rotor
 u_w is the non-uniform wake velocity (i.e. a function of the distance and direction from the hub)

Investigations made in connection to the validation of the wake models implemented showed, that using linear combination of wind speeds or using exponents of order 3 only gave marginal differences on the averaged wind speed. The integration in (1) is done by numerical means.

Ad. 2: Wake Combination Models

This averaging may be done in a variety of combinations. Djerf [2] states on option of four different wake combination methods: 1) Sum of squares of velocity deficits, 2) Energy balance, 3) Geometric sum, 4) Linear superposition. According to Djerf it is not recommend using methods (3) and (4). Schepers [3] suggests another approach. Schepers first calculates the wake from the upstream turbine. Then this wake is used for calculating the axial force coefficient on the second turbine downstream. The initial velocity deficit behind the second turbine is then calculated from the axial force.

In WindPRO, the ‘Sum of squares of velocity deficit’ methodology is used.

Sum of Squares of Velocity Deficits

The N.O. Jensen model initially implemented in the WindPRO Park module as well as the WAsP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution.

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2} \quad (2)$$

where δV is the velocity deficit defined as $(1-V/U)$ – where U is the free wind speed
 n is the number of upstream turbines

Lange et. al [1] uses a slightly different formulation of the sum of the squares of velocity deficits approach. This equation is used in conjunction with equation (1) to calculate the deficit.

$$(u_0 - u_c)^2 = u_0^2 \left(1 - \frac{u_c}{u_0} \right)^2 = \frac{1}{A} \int_{rotor} \sum_{i, allwakes} (u_{rotor(i)} - u_{w(i)})^2 dA \quad (3)$$

Outline of the calculation procedure

An overview of the calculation procedure is as follows:

1. The calculation is initiated with the turbine positioned at the most upstream position (lup turbine)
2. Find (calculate) the wind speed directly upstream of the turbine
3. Calculate the wind speeds downstream of this turbine, i.e. for all downwind turbine positions
4. Calculate the deficits for all downstream turbine positions, i.e. relating to the free wind speed
5. If the downstream turbine is in a partial wake, then reduce the velocity deficit with the fraction of the overlap area to the rotor area of the downstream turbine.
6. Calculate the square of the velocity deficits
7. Continue with the next turbine (using step 1), by summing the squares of the velocity deficits.

References

- [1] Lange, Bernard et al: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLAP*, Wind Energy 2003; 6: 87-104
- [2] Djerf, E. & Mattson, H.: *Evaluation of the Software program WindFarm and Comparisons with Measured Data from Alsvik*, FFA TN-2000-30,
http://www.vindenergi.foi.se/Rapporter/TN2000_30_WindFarm.pdf
- [3] Schepers, *ENDOW: Validation and Improvement of ECN's wake model*, ECN-C-03-034.

6. Introduction to Turbulence and Wakes

Wind turbines operating in wakes are subjected to higher turbulence levels than turbines operating in the free wind, thus appropriate turbulence calculations should be made before selecting the proper turbine design class when having clusters of turbines. This is due to the fact that the fatigue loads and possibly also the extreme loads are higher when the turbulence levels increases.

The wake added turbulence may be calculated using different wake or turbulence models. These models are typically very different in detailing level – and possible also in accuracy. The models range is from simple engineering models to the more advanced computational fluid dynamic (CFD) models. The CFD-models are typically also very demanding in terms of calculation time.



This chapter gives an introduction to the calculations and operations performed on the measured ambient turbulence data, and how the turbulence data from single wake models is merged. We also give a brief introduction to the turbulence calculation required by the IEC 61400-1 structural code.

Figure 1: Part of a Flash Animation
Created using WindPRO.

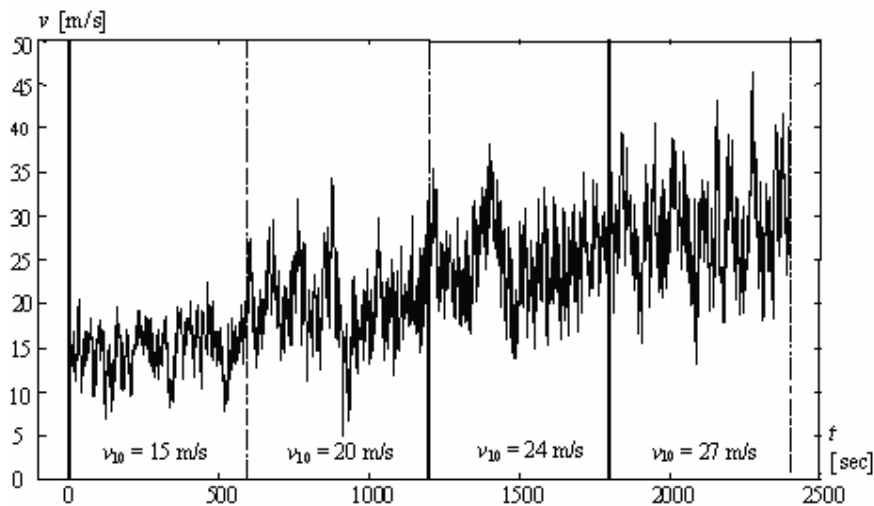


Figure 2: Turbulent winds – simulated at different mean wind speeds, from [1].

WindPRO contains several models for calculating the wake added turbulence. The ambient turbulence level must be user specified, e.g. through measured data or alternatively through the roughness classification.

The Turbulence Calculation

When estimating the design, lifetime and fatigue on wind turbines, the turbulence levels are of outmost importance. The turbulent winds arise from several sources:

1. Orography induced turbulence, i.e. flow over hills and mountains
2. Roughness induced turbulence, i.e. flow generated by objects within the landscape
3. Turbine generated turbulence, i.e. turbulence in the wake of the turbines
4. Obstacle induced turbulence, i.e. turbulence generated in the wake of large nearby obstacles

The turbulence intensity is defined as the ratio between the standard deviation of the wind speed, σ_u , and the 10-minute mean wind speed, U_{10} . When dealing with wind turbine wakes, it is tradition to relate the 10-minute mean wind speed to the free wind speed, i.e. the wind speed outside the wake.

$$I_T = \frac{\sigma_u}{U_{10}} \quad (1)$$

The current edition of WindPRO – version 2.5 – deals primarily with the turbine generated turbulence. Orography and roughness generated turbulences are included only through the on-site meteorological measurements – or alternatively through user-defined turbulence input levels.

Estimating the Ambient Turbulence Level

When doing the wake calculations then the ambient turbulence level must be estimated either through on-site meteorological measurements, through simple roughness classifications or using numerical flow models. Even if the definition of the ambient turbulence seems reasonable simple (see equation 1), then the estimation of this turbulence intensity is quite difficult due to the stochastic nature of the turbulence, i.e. for a given wind speed then measurements of turbulence intensity will show significant scatter. This scatter can be modelled only accurately as a random variable; so as a minimum requirement it is recommended not only to calculate the mean turbulence level, but also the standard deviation of the turbulence intensity. Actually, this is done automatically when you load meteorological measurements in a meteo-object in WindPRO.

In WindPRO three different measures of the ambient turbulence is used and calculated in each bin (wind speed and sector):

1. Mean (average) turbulence
2. Standard deviation of turbulence
3. Representative (characteristic) turbulence

The first two measures are purely statistical estimators, used in order to describe the turbulence distribution. The last issue (3) is included as the structural codes typically require that a design value of turbulence is used; i.e. the representative turbulence is some function of the mean and standard deviation of the turbulence. Actually, this definition of the representative ambient turbulence levels varies also with different structural codes, e.g. the IEC 61400-1 second and third editions [2, 3] have different definitions of this parameter, see below.

Ambient Turbulence Level According to the IEC 61400-1 second edition

When estimating the wind condition to check if an IEC class turbine is suitable for a particular site, then the IEC 61400-1 second editions calls for calculating an I_{15} parameter which is a characteristic value of hub height turbulence intensity at 10 min average wind speed of 15 m/s. The characteristic value is calculated by adding the measured standard deviation of the turbulence intensity to the measured or estimated mean value (only considering the 15 m/s bin values), i.e.

$$I_{15} = \mu_{I|15m/s} + 1.0 * \sigma_{I|15m/s} \quad (2)$$

The IEC 61400-1 ed. 2 requires the I_{15} parameter to be estimated using statistical techniques applied to wind speeds and turbulence measurements above 10 m/s. It specifies also, that the influence of the wakes should be accounted for.

Ambient Turbulence Level According to the IEC 61400-1 third edition

IEC 61400-1 ed. 3 has a slightly different approach to turbulence modelling as it focuses on the standard deviation, σ_u , rather than the turbulence intensity. The IEC 61400-1 ed. 3 requirement is that the following equation is fulfilled for all wind speeds from 0.6 times the rated wind speed to the cut-out wind speed:

$$\sigma_1 \geq I_{eff} \cdot V_{hub} + 1.28 \cdot \hat{\sigma}_\sigma \quad (3)$$

where σ_1 is the turbulence standard deviation from the normal turbulence model as specified in the IEC code
 I_{eff} is the total turbulence (ambient and wake)
 V_{hub} is the wind speed at hub height level
 σ_σ is the measured standard deviation of the turbulence standard deviation

The factor 1.28 is applied because a 90% percentile is sought.

Calculating the Ambient Turbulence from Measurements

When on site measurements are available then WindPRO is able to calculate the mean turbulence intensity table as well as the standard deviation of the turbulence intensity and a user defined representative turbulence level.

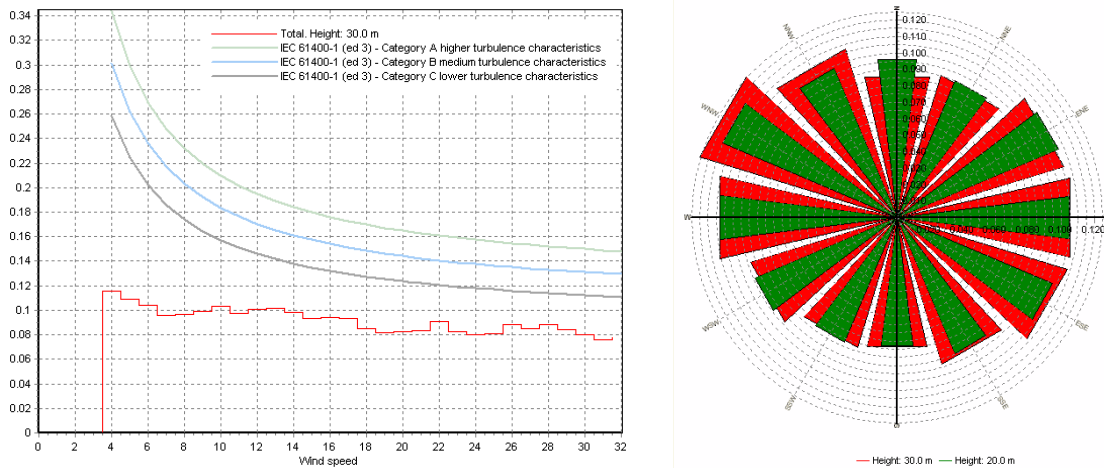


Figure 3: Measured Ambient Turbulence Levels.

Calculating Turbulence Intensity from Roughness Data and/or Roughness Maps

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain – be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [4]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[\frac{1}{\ln[z / z_0]} \right] \quad (4)$$

The value of A is reported to vary approximately between 2.5 to 1.8. κ is the Von Karman constant, which is equal to 0.4. In DS 472 [5] the product between A_x and κ is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC 61400-2, the characteristic data needed is actually a mean value plus one standard deviation, so some estimate of the standard deviation of the turbulence is needed.

A Rule of Thumb to Estimate the Standard Deviation of the Turbulence

This standard deviation may be estimated – from a rule of thumb widely used in Germany. In the general case, the coefficient of variation ($COV = \sigma / \mu$) is set to 20%. Only for forest sites and for extreme hill tops, this value is not sufficient but must be replaced by measurements.

Vertical Scaling of the Ambient Turbulence Level

Often, when turbulence measurements are available from the site, the measurements are not taken at hub-height level. This calls for a vertical scaling of the ambient turbulence, which is done by assuming homogeneous terrain (an approximation to the real nature). Preferably, the turbulence should be taken from hub-height measurements.

Assuming that the wind flow is a horizontally homogeneous (i.e. the properties of the flow do not change in the horizontal direction), then the standard deviation of the wind speed process is only depended of the height above the terrain, z .

The turbulence intensity in the height x meters is defined as:

$$IT(x) = \frac{\sigma_U(x)}{U_{10}(x)} \quad (5)$$

Where

IT is the turbulence intensity

σ_U is the standard deviation of the wind speed

U_{10} is the mean wind speed averaged over 10 minutes

Experimental data has shown that the standard deviation of the wind speed only decreases very slowly. In Armit [6] & Dyrbye & Hansen [7], it is said, that it is reasonable to use constant standard deviations up to about the half-height of the internal boundary layer. This assumption is also used in WAsP and in most structural codes.

Using this assumption, the vertical scaling of turbulence intensity between two heights is simply calculated by assuming the same standard deviations in the two heights (x and y meters or feet).

$$\begin{aligned} \sigma_U(x) &= \sigma_U(y) && \Leftrightarrow \\ IT(x) \cdot U_{10}(x) &= IT(y) \cdot U_{10}(y) && \Leftrightarrow \\ IT(y) &= \frac{U_{10}(x)}{U_{10}(y)} IT(x) \end{aligned} \quad (6)$$

So now the problem is reduced into calculating the mean wind speed in the new height. The vertical scaling of wind speeds may be done using the power law vertical wind profile a purely empirical equation. The power law wind profiles also require quite homogenous terrain.

$$U_{10}(y) = U_{10}(x) \cdot \left[\frac{y}{x} \right]^\gamma \quad (7)$$

where

γ is the wind gradient exponent

The wind gradient exponent is known to be very depended on the roughness length or the roughness class. The table below gives guidelines for selecting the wind gradient exponent – if no measured data is available:

Roughness Class	Roughness Length	Wind Gradient Exponent
0	0.0002	0.1
1	0.03	0.15
2	0.1	0.2
3	0.4	0.3

Inserting the equation (4) into (3) we obtain the turbulence scaling law, valid for homogeneous terrain:

$$IT(y) = \frac{U_{10}(x)}{U_{10}(y)} IT(x) = IT(x) \cdot \left[\frac{y}{x} \right]^{-\alpha} \quad (8)$$

Turbulence from Wind Turbine Wakes

The wake added turbulence is either derived from the (single) wake models that include turbulence modelling or from dedicated (empirical) turbulence models. The turbulence calculated from the different models may be parameterized in numerous ways, see e.g. Figure 4 which holds output from the eddy Viscosity wake model. Using the EV-model one may relate the eddy viscosity to the turbulence intensity or alternative use empirical values. Again, other models have wake turbulence included in a purely empirical manner. The turbulence model must be used in connection with a wake model – in order to take the reduced wind speeds in the wind farm into account.

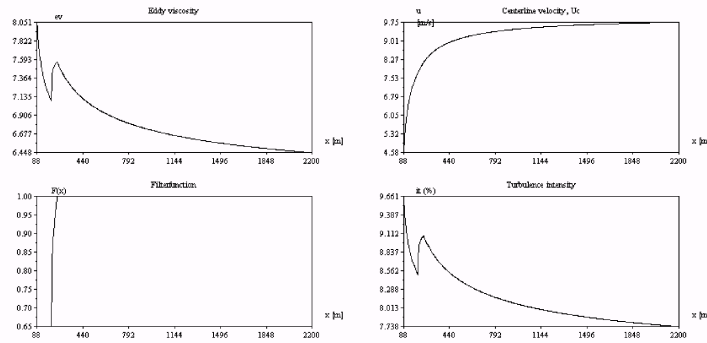


Figure 4: Single Wake Turbulence Modelling using the Eddy Viscosity Model.

The results from the turbulence models – may typically come within one of four categories:

1. Added turbulence model – calculated for the wake after a single turbulence
2. Added turbulence model – calculated for all surrounding turbines
3. Total turbulence model - calculated for the wake after a single turbulence
4. Total turbulence model – calculated for all surrounding turbines

Models (1) and (2) give the wake added turbulence contribution. This should be added to the ambient turbulence level. The model type (3) gives the total turbulence level for a given wake at a given position (ambient and wake added), and this must be summed into a combined effect considering all upstream

turbines. The model type (4) gives the total turbulence level in an integrated manner, thus no single wake adding is needed.

All of the turbulence models implemented in WindPRO belong to any of these four types.

Calculating the wake added turbulence intensity

The turbulence intensity is defined as the ratio of standard deviation to the mean wind speed. It is common practice to relate the turbulence intensity – also within the wake – to the ambient free wind speed. Also, it is practice to assume that the added turbulence level may be added as independent stochastic variables.

In the Danish Recommendation [6] the total turbulence intensity is actually calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (9)$$

Partial Wakes - Turbulence

When the turbine operates in a partial wake, we use the equation (6) to calculate the added turbulence level – considering the rotor area with ambient turbulence only. A linear weighting with rotor areas is assumed.

Converting From Time Series Turbulence to Turbulence Tables

From WindPRO 2.5, the meteo object is the container for three different turbulence tables: The representative or characteristic turbulence table, the mean turbulence table and the standard deviation turbulence table. Each of these tables is used to store the turbulence intensities.

Each table with turbulence intensities is typically binned with an angular interval equal to 30 degrees and a wind speed interval equal to 1.0 m/s. In each bin the sample statistics are then calculated (mean and standard deviation), see also [8]:

$$\overline{X} = \frac{X_1 + X_2 + \dots + X_n}{n} \quad (10)$$

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \overline{X})^2}{n-1}} \quad (11)$$

where \overline{X} is the sample mean
 S is the sample standard deviation

These two sample statistics are stored in the mean and standard deviation tables respectively. The representative turbulence table values is calculated using the IEC code relations (or user defined relations) as indicated earlier in this chapter. Not only are the binned statistics stored and presented but actually also omni-directional statistics and the sector-wise results. All of these statistics are – as a default setting – derived directly from the time series data.

Manual Editing of the Mean and Standard Deviation Turbulence Tables

When you choose to manually edit the turbulence tables (mean + standard deviation tables), then the omni-directional and sector-wise means are calculated using the assumption that the binned sample distributions are independent Gaussian distributions.

By using this model to calculate statistics you will typically have results that is only differing a few per mille when compared to statistics based on the measured time series data. This is due to the fact that the samples may not fit perfectly to the Gaussian distribution.

Note: Only the mean + standard deviation tables may be edited. The representative turbulence is calculated automatically based on this table.

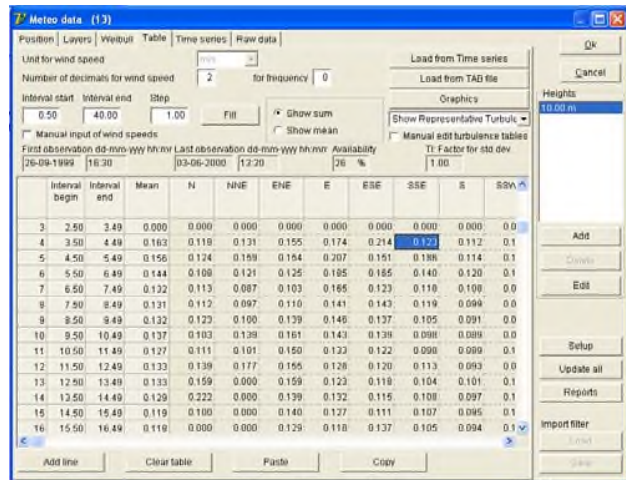


Figure 5: Screen Shot of Meteo Object Turbulence Table in WindPRO 2.5.

If you wish to reset your manual edits, then you must reload tables from the time series data.

We calculate the statistics using a Monte Carlo simulation approach. This approach requires both lookup in the frequency table (to get the number of actual samples in each bin) and lookup in the binned turbulence sample statistics (mean and standard deviation). The approach follows the following procedure, as outlined in this pseudo code algorithm:

```

for each sector-bin (typically 1 - 12) do
  Get mean and standard deviation of selected sector-bin
  Make Gaussian distribution using the mean and standard deviation
  Lookup in frequency table to find number of occurrences (cnt) in this bin
  If wind speed is less than the 'include turbulence wind speed' then cnt = 1
  If cnt = 0 then we assume that cnt = 1
  Use Gaussian distribution to simulate 'count' new occurrences
  Update omni-directional Statistics using the simulated data
end

```

Calculate the omni-directional mean and standard deviations

Wind Speed Interval [m/s]		Omni-directional Turbulence Intensity		
From	To	Time series	Gaussian model	Difference
6.5	7.5	0.132	0.132	0.000
7.5	8.5	0.131	0.132	0.001
8.5	9.5	0.132	0.132	0.000
9.5	10.5	0.137	0.137	0.000
10.5	11.5	0.127	0.127	0.000
11.5	12.5	0.133	0.133	0.000
20.50	21.50	0.099	0.099	0.000
21.50	22.50	0.114	0.114	0.000
22.50	23.50	0.097	0.098	0.001
23.50	24.50	0.093	0.093	0.001
24.50	25.50	0.092	0.097	0.005
25.50	26.50	0.099	0.097	0.002

Table 1: Comparing Selected Results from Turbulence Table Calculations.

A sample calculation – where the representative turbulence has been extracted using a factor on the standard deviation equal to 1.00 - is shown in the Table 1. Please note, that the difference between representative turbulence calculated using the time series data and the Gaussian model data increases when the frequency decreases (typically at very rare bins in the upper tail of the distribution).

References

- [1] Misfeldt & Thøgersen, *Reliability Analysis of Wood Structures*, Aalborg University, 1997 (unpublished)
- [2] IEC 61400-1:1998 (2nd edition)
- [3] IEC 61400-1 (3rd edition)
- [4] DNV & Risø: *Guidelines for the Design of Wind Turbines*, Risø National Laboratory
- [5] DS472, Danish Code of Standards for Wind Turbine Structures
- [6] Armit, *Wind Structures*, Lecture Series, Von Karman Institute for Fluid Dynamics, 1976.
- [7] Dyrbye & Hansen, *Wind Loads on Structures*, John Wiley and Sons, 1996
- [8] Sheldon M. Ross: *Introduction to Probability and Statistics for Engineers and Scientists*, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons, 1987

7. Danish Recommendation – Turbulence Model

The Danish Recommendation [1] from 1992 specifies a quite simple wake added turbulence model. If the turbines are erected in a cluster with a minimum distance between the turbines of 5 times the rotor diameter – or in a row with the distance 3 times the rotor diameter – then a added turbulence intensity of $I_{\text{park}} = 0.15$ can be used. An alternative is to use the a mean-contribution, which varies by the mean wind speed and the distance between the turbines:

$$I_{\text{park}} = \beta_v \cdot \beta_l \cdot 0.15 \quad (1)$$

where β_v is a parameter taking the mean wind speed into account (see the Figure 1)

β_l is a parameter taking the distance between the turbines into account (Figure 2 and 3).

The β_l parameters are dependent on the geometrical configuration of the wind farm, i.e. if the wind farm is erected in a cluster (Figure 2) or in a row (Figure 3).

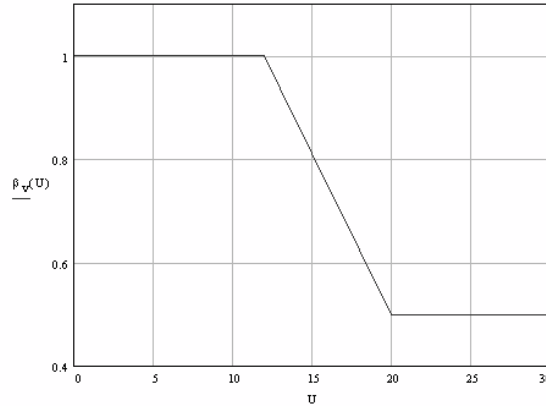


Figure 1: Factor taking wind velocity into account, β_v .

The β_l factor is determined from the Figures 2 and 3.

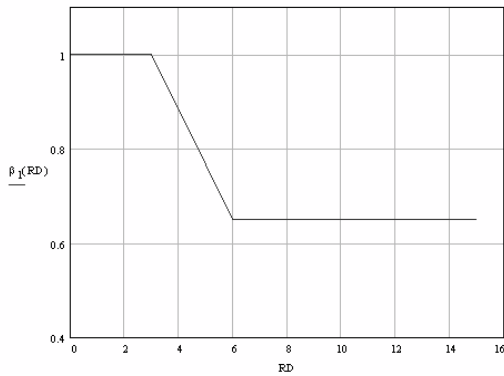


Figure 2: β_l for turbines in a row.

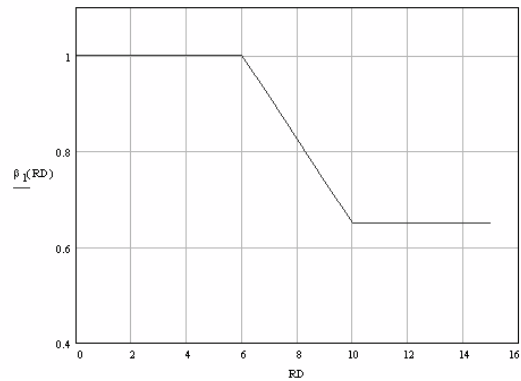


Figure 3: β_l for turbines in a cluster.

The Wake Added Turbulence

The total turbulence intensity is calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (2)$$

References

[1] *Recommendation for the fulfillment of the requirements found in the technical criteria*, Danish Energy Agency, 1992

8. Turbulence Model – Frandsen & DIBt

S. Frandsen and M.L. Thøgersen [1] report an empirical turbulence model for calculating the integrated wake effect of turbines. This model takes into account the different structural fatigue responses of the structural materials considered, e.g. steel in the towers and hub extenders and glass fibre reinforced polyester (GRP) or glass fibre reinforced epoxy (GRE) in the blades. The equations below assume that the wind direction is approximately uniform distributed. Reference is made to Frandsen & Thøgersen [1] and Guidelines for the Design of Wind Turbines [2].

This model is included as a recommended model in the German DIBt Richtlinie [3].

Determining the Total Turbulence Intensity

The total turbulence intensity is determined from:

$$I_{T,total} = \left[(1 - N \cdot p_w) I_T^m + p_w \sum_{i=1}^N I_{T,w}^m(s_i) \right]^{1/m} \quad (1)$$

$$I_{T,w} = \sqrt{\frac{1}{[1.5 + 0.3 \cdot s_i \cdot \sqrt{v}]^2} + I_T^2} \quad (2)$$

where $p_w = 0.06$ (probability of wake condition)

$s_i = x_i / RD$

N is the number of closest neighboring wind turbines

m is the Wöhler curve exponent of the considered material

v is the free flow mean wind speed at hub height

x_i is the distance to the i -th turbine

RD is the rotor diameter

I_T is the ambient turbulence intensity (free flow)

$I_{T,w}$ is the maximum turbulence intensity at hub height in the center of the wake

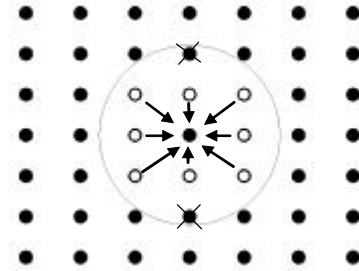
The number of closest neighboring turbines is determined as follows
– see also the figure to the right.

$N=1$: 2 wind turbines

$N=2$: 1 row

$N=5$: 2 rows

$N=8$: Wind farms with more than two rows



Increased Turbulence in Very Large Wind Farms

If the wind farm has more than five rows, the wind farm itself heavily influences the ambient wind climate. Also – if the distance between turbines in rows perpendicular to the predominant wind direction is less than 3 times the rotor diameter, an increase in mean turbulence level must be taken into account. This is done by substituting the ambient turbulence levels in (1) and (2) with the turbulence calculated from the equations (3) and (4).

$$I_T^* = 0.50 \cdot \sqrt{I_w^2 + I_T^2} + I_T \quad (3)$$

$$I_w = \frac{0.36}{1 + 0.08\sqrt{s_r s_f} v} \quad (4)$$

where $s_r = x_r / RD$
 $s_f = x_f / RD$
 s_r is the distance *within* the row
 s_f is the distance between rows

References

- [1] S. Frandsen & M.L. Thøgersen, *Integrated Fatigue Loading for Wind Turbines in Wind Farms by Combining Ambient Turbulence and Wakes*, Wind Engineering, Volume 23, No. 6, 1999.
- [2] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition.
- [3] Deutsches Institut für Bautechnik – DIBt, *Richtlinie für Windenergieanlagen, Einwirkungen und Standsicherheitsnachweise für Turm und Gründung*, Fassung März 2004.
- [4] S.T.Frandsen, *Turbulence and turbulence generated structural loading in wind turbine clusters*, Risø National Laboratory, January 2007.

9. Turbulence Model – D.C. Quarton & TNO Laboratory

A simple equation to determine the wake added turbulence has been proposed by D.C. Quarton and J.F. Ainslie [1]. The parameters in the equation have been re-calibrated by Quarton and Ainslie (the modified values) and also the Dutch TNO laboratory [2].

The main form of the equation is

$$I_{add} = K_I \cdot C_T^{\alpha_1} \cdot I_{amb}^{\alpha_2} \cdot (X / X_n)^{\alpha_3} \quad (1)$$

where K_I is a proportionality constant

$\alpha_1, \alpha_2, \alpha_3$ are exponents

X is the downstream distance (in meters)

X_n is a characteristic wake length (either denoted near wake or far wake)

I_{amb} is the ambient turbulence

The near wake length (X_n) is determined as described in the chapter dealing with the eddy viscosity wake model. In case of the TNO model, then the near wake length is replaced with a slightly different expression for the far wake length, see [2].

The proportionally constant and exponents are determined from the table below

Reference	K_I -Constant	α_1 -exponent	α_2 -exponent	α_3 -exponent
Quarton and Ainslie (original)	4.800	0.700	0.680	-0.570
Quarton and Ainslie (modified)	5.700	0.700	0.680	-0.960
Dutch TNO laboratory	1.310	0.700	0.680	-0.960

Note, that the ambient turbulence must be entered in percent (i.e. 10) when using the Quarton-Ainslie constants, while the TNO-constants are with ambient turbulence as decimal number (i.e. 0.10).

At the Figure 1 it is also easily seen, that the two models (Ainslie (modified) and TNO) actually are the same.

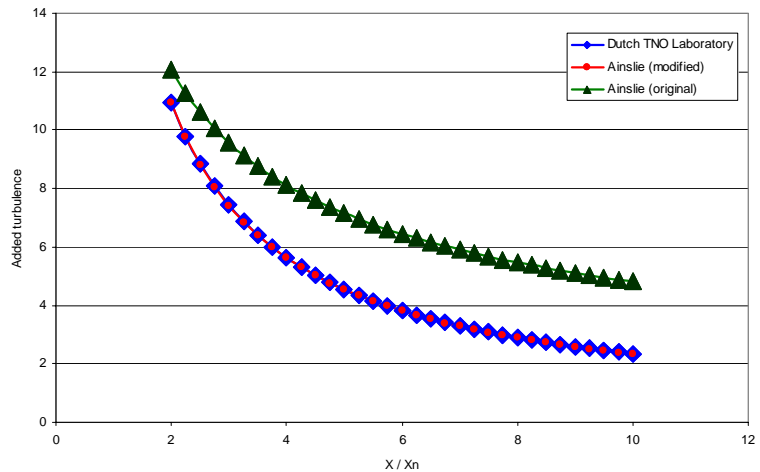


Figure 1: Wake Added Turbulence from the Three Models.

References

- [1] Quarton and Ainslie, *Turbulence in Wind Turbine Wakes*, Wind Engineering Vol 14 No 1
- [2] *European Wind Turbine Standards II*, ECN-C--98-096, December 1998.

10. Turbulence Model – B. Lange

The B. Lange turbulence model can only be used with the eddy viscosity wake model, because the turbulence parameters are derived directly from the eddy viscosity.

Turbulence within the Wake

The turbulence intensity, I_T , is defined as the standard deviation of the wind speed process divided by the mean wind speed, i.e.

$$I_T = \sigma_u / u_0$$

It is possible to relate the eddy viscosity to the turbulence intensity. According to Lange et al [1], the turbulence intensity within the wake can be calculated using the following relation below. Please note that the equation relates to the free wind speed, U_0 :

$$I_T = \varepsilon \frac{2.4}{\kappa \cdot U_0 \cdot z_h}$$

Alternative Empirical Approach

Another alternative empirical characterization of the wake turbulence was proposed by Quarton and Ainslie [2]. Their equation is based on a parameterization on the near wake length – which is primarily used in relation with the Eddy Viscosity model. They report, that the empirical turbulence decay is somewhat higher than other model predictions. The equation is:

$$I_{add} = 4.8 C_T^{0.7} I_{amb}^{0.68} [X / X_n]^{-0.57}$$

where I_{add} is the added turbulence intensity from the wind turbine wake
 I_{amb} is the ambient wind speed
 X is the downstream distance
 X_n is the near wake length

This alternative approach can also be used with other wake modes, as the near wake length is easily determined through empirical equations. For further details on the near wake length – please see the chapter on the Eddy viscosity wake model.

References

- [1] Lange, Bernard; H.P. Waldl; A.G. Guerrero; D. Heinemann & R.J. Barthelmie: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003 6:87-104.
- [2] Quarton & Ainslie: *Turbulence in Wind Turbine Wakes*, Wind Engineering, Volume 14, No. 1.

11. Turbulence Model – G.C. Larsen

The G.C. Larsen is a simple empirical equation to determine the turbulence level within the wake. Reference is made to the paper ‘*Wind Field in Wakes*’ [1] and the European Research Project – European Wind Turbine Standards - EWTS II [2].

Turbulence Intensity

At positions downstream of the turbine, the wake added turbulence intensity can be determined from the equation:

$$I_w = 0.29S^{-1/3} \sqrt{1 - \sqrt{1 - C_T}} \quad (1)$$

where S is spacing expressed in rotor diameters
 C_T is the thrust coefficient

The expression for turbulence intensity is only valid for distances larger than two rotor diameters downstream.

References

- [1] G.C.Larsen, J. Højstrup, H.A. Madsen, *Wind Fields in Wakes*, EUWEC '96, Gothenburg, 1996.
- [2] *European Wind Turbine Standards II*, ECN-C-99-073, 1999

12. User guide to wake modeling and turbulence calculation

The new wake models are operated from the PARK calculation exactly like previous versions of WindPRO. It is possible to operate the PARK calculation with exactly the same wake model as before. The new models offer alternatives to the standard N.O. Jensen model and provide the possibility to calculate the wake-induced turbulence in the wind farm. EMD recommends using the N.O. Jensen model as the standard model unless special needs require the use of the alternative models.

PARK default settings

When the PARK calculation option is started WindPRO default settings applies. These make the calculation identical to previous versions of WindPRO (WindPRO 2.4 mode). The settings are as shown below.

The screenshot shows the 'WindPRO' software interface with the 'Wind distribution' tab selected. The 'Name' field is 'As a new wind farm with local data'. The 'Air density varies at WTGs (here: 395,0 m)' is set to 1,199. The 'Hub height for key results' is 50,0 m. The 'Report: Additionally to result also reduce energy with:' is 10 %. The 'Report: Label for reduced energy result:' is 'Result'. The 'Calculate:' section has 'Energy reductions due to wakes' checked. The 'Wake model' section has 'N.O. Jensen (RISØ/EMD)' selected. The 'Turbulence model' section has 'No model (ambient turbulence only)' selected. The 'Sector wise Parameters' section shows 'Terrain: User defined, Tt: 14,7 %, WDC: 0,075, sectors: 1.'

The N.O. Jensen (RISØ/EMD) model is selected with a Wake Decay Constant of 0,075 uniformly for all sectors. This is the wake model used in WindPRO 2.4. It does not allow calculation of wake-induced turbulence, but it ensures that the PARK result is identical to earlier calculations.

The only options available are the Wake calculation settings and the Sector wise parameters.

The Wake calculation settings allow the user to modify the basic parameters of the wake calculation. They are common to the other models. The start, end and step of wind speed and angle are set to cover the full range at a reasonable level of detail and should preferably not be changed.

General calculation settings	
Description	Value
Start wind speed [m/s]	0,50
End wind speed [m/s]	30,50
Step in wind speed [m/s]	1,00
Start angle [deg]	0,5
End angle [deg]	360,0
Step in angle [deg]	1,0

Ok Cancel

The sector wise parameters are covered in a separate section.

N.O. Jensen (EMD) : 2005

As described in theory this model is identical to the old N. O. Jensen model except that it allows the calculation of wake induced turbulence, reduced wind speeds inside wind farm and a park power curve based on the PPV model

Main | WTGs | Wind distribution | Description |

Name: As a new wind farm with local data

Air density varies at WTGs (here: 395,0 m): 1,199 Edit/View ☒ Auto / Individual

Hub height for key results: 50,0 m 50 meters recommended

Report: Additionally to result also reduce energy with: 10 % (e.g. uncertainty and/or loss reduction)

Report: Label for reduced energy result: Result Label: Result-10,0%

Calculate: ☒ Energy reductions due to wakes
☐ Wake added turbulence
☐ RIX (Ruggedness Index)
☐ Reduced wind speeds inside wind farm
☐ Park power curve based on PPV model

Wake model

- ☐ No model
- ☐ N.O. Jensen (RISØ/EMD)
- ☒ N.O. Jensen (EMD) : 2005
- ☐ Eddy Viscosity Model (J.F. Ainslie) : 1986
- ☐ EWTS II (G.C.Larsen) : 1999

Turbulence model

- ☒ B.Lange: 2002 - Eddy viscosity turbulence
- ☐ Danish Recommendation: 1992
- ☐ G.C.Larsen / EWTS II : 1999
- ☐ S. Frandsen: 1999 - Effective turbulence model
- ☐ Quarton/Ainslie: 1989 - Empirical turbulence model
- ☒ Empirical turbulence - Dutch TNO laboratory : 2004
- ☐ DIBt: 2004 - Added Turbulence Model

Advanced model parameters Wake calculation settings Turbulence model parameters

Sector wise Parameters

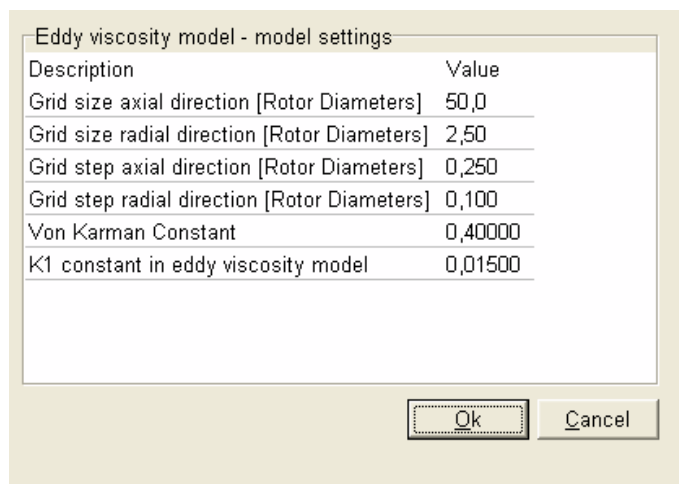
Terrain: User defined, Tt: 14,7 %, WDC: 0,075, sectors: 1. Edit Edit WAsP parameters

Ok Cancel

EMD recommends using the Empirical turbulence – Dutch TNO laboratory turbulence model together with N. O. Jensen, but it can be combined with the others except for B. Lange: 2002.

Eddy viscosity model

Selecting the eddy viscosity model by Ainslie 1986 enables the same options as N. O. Jensen except that it is now possible to use the B. Lange turbulence model and that a set of advanced model parameters can be selected.



Description	Value
Grid size axial direction [Rotor Diameters]	50,0
Grid size radial direction [Rotor Diameters]	2,50
Grid step axial direction [Rotor Diameters]	0,250
Grid step radial direction [Rotor Diameters]	0,100
Von Karman Constant	0,40000
K1 constant in eddy viscosity model	0,01500

Ok Cancel

The parameters have primarily to do with the grid size of the calculation. Smaller grid size means a slower calculation and since it is already a slow calculation we recommend that these be not reduced any further unless a special need requires this.

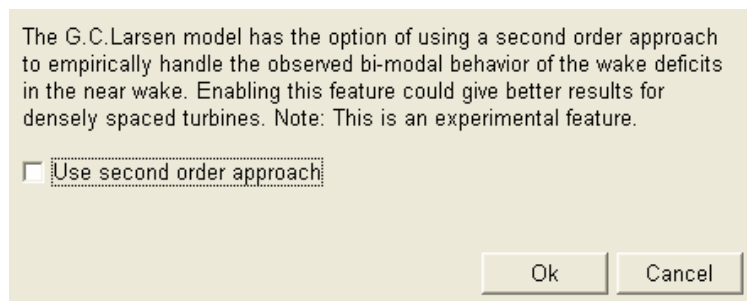
Von Karmans constant is a well-described constant and should not be changed.
For the K1 constant please refer to the theoretical section.

EWTS II

The EWTS II model allows the same turbulence options as the N. O. Jensen model but the recommended turbulence model is the G. C. Larsen/EWTS II model.

A special “Advanced model parameter” with this model gives the possibility to use a second order approach.

As it is an experimental feature it should be used with caution. Please refer to the theoretical section.



The G.C.Larsen model has the option of using a second order approach to empirically handle the observed bi-modal behavior of the wake deficits in the near wake. Enabling this feature could give better results for densely spaced turbines. Note: This is an experimental feature.

☐ Use second order approach

Ok Cancel

Sector wise parameters

These are the parameters defining the ambient turbulence. As mentioned above the default setting will just give a uniform wake decay constant of 0,075 which is suited for most sites (see below).

Grey columns are needed parameters for the selected wake model. Ambient turbulence is scaled in height according to description in the WindPRO manual.


Height for turbulence specification [m]

Number of wake sectors


From [deg]	To [deg]	Terrain type and class	Turbulence intensity	Roughness length [m]	Wake decay constant
-180	180	User defined	0,1468	0,055	0,075

Cancel


Turbulence Intensity



Roughness Length



Wake decay constant



Turbulence, roughness length and wake decay constant are all linked. The roughness length is (part of) what causes the turbulence and it is the turbulence that gives the wake decay constant.

These three parameters can be changed individually or set altogether by selecting a terrain type.

A more detailed definition of the ambient turbulence can be defined by adding sectors. The three circle diagrams on the right will then show the directional distribution of the turbulence, roughness and wake decay constant.

Another option is to load turbulence data from the Meteo object. This requires a meteo object with a time series of turbulence intensity (typically generated from standard deviations of 10-minute readings).

Pressing the “Load from Meteo Data” button opens a selection tool where the proper meteo object and height can be selected. The user can then select only to include turbulence for specific wind speeds or simply import turbulence for the full range of wind speeds. If more wind speeds are selected WindPRO will make an average of the turbulence intensity to calculate the appropriate roughness lengths and wake decay constants.

With Ok the data are loaded and presented as shown below.

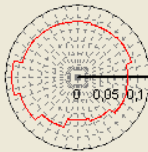
Grey columns are needed parameters for the selected wake model. Ambient turbulence is scaled in height according to description in the WindPRO manual.

Height for turbulence specification [m]

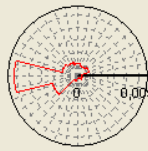
Number of wake sectors

From [deg]	To [deg]	Terrain type and class	Turbulence intensity	Roughness length [m]	Wake decay constant
-15	15	User defined	0,0925	0,001	0,0444
15	45	User defined	0,0908	0,0008	0,0434
45	75	User defined	0,0923	0,001	0,0443
75	105	User defined	0,0828	0,0003	0,0405
105	135	User defined	0,0849	0,0004	0,041
135	165	User defined	0,0748	0,0002	0,04
165	195	User defined	0,0709	0,0002	0,04
195	225	User defined	0,0856	0,0004	0,0412
225	255	User defined	0,0995	0,0022	0,0507
255	285	User defined	0,1091	0,0052	0,0531
285	315	User defined	0,0949	0,0013	0,0462
315	345	User defined	0,0938	0,0012	0,0453

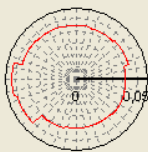
Turbulence Intensity



Roughness Length



Wake decay constant



The window also has a calculator that can be used to calculate the turbulence intensity based on the surface roughness.

Ambient turbulence estimation

The relation between the turbulence and surface roughness can - in the case of homogeneous terrain - be derived from boundary layer theory. Note that the relation is a very rough idealization of the true stochastic nature of turbulence. Experiments have shown that the value of A varies between 1.8 and 2.5. The von Karman constant has the value of 0.4.

$$I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[\frac{1}{\ln[z/z_0]} \right]$$

Ax:

kappa:

z:

z0:

Result:

With this tool it is possible to calculate “manually” the relation between roughness length and turbulence intensity. Please note that the parameter A is an empirical size, which is not exact.

The turbulence models

The turbulence models are described in detail in the theoretical section. The operation of them in WindPRO is almost identical, the only difference is some special parameter settings that some of them facilitates and the time it takes to calculate them. The choice of model does not influence the format of the printout beyond the result they provide and the mentioning of the model and parameters used.

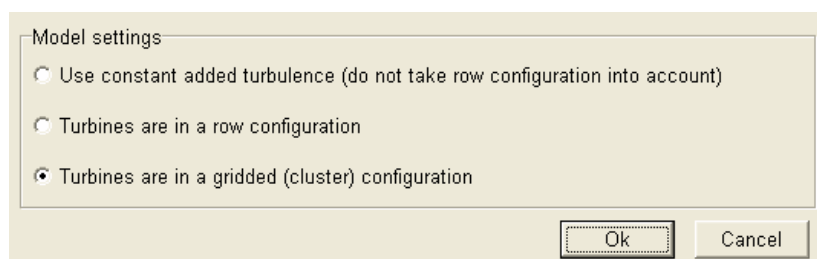
The following is a summary of the possible parameter settings.

B. Lange

There are no special parameters for this model. The input data comes from the Eddy viscosity wake model.

Danish Recommendation

The options available for this model are shown below. Please refer to the theoretical section for an explanation of the parameters.



Model settings

☐ Use constant added turbulence (do not take row configuration into account)

☐ Turbines are in a row configuration

☒ Turbines are in a gridded (cluster) configuration

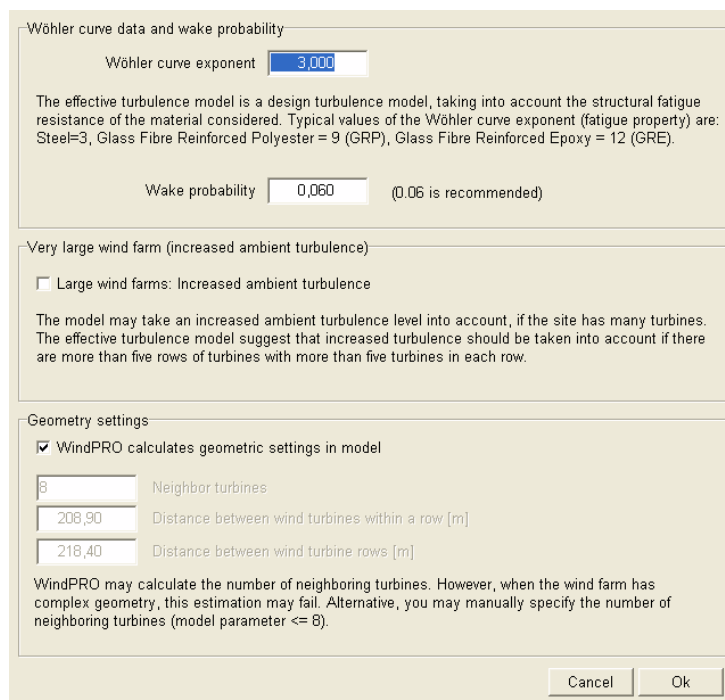
Ok Cancel

G. C. Larsen / EWTS II

There are no special options for this model.

S. Frandsen, 1999

For this model there are a number of options. These are all explained in the theoretical part. A special feature is the geometrical section. WindPRO should in normal cases be able to figure this out by it self, but with random layout wind farms, this may not be done correctly.



Wöhler curve data and wake probability

Wöhler curve exponent

The effective turbulence model is a design turbulence model, taking into account the structural fatigue resistance of the material considered. Typical values of the Wöhler curve exponent (fatigue property) are: Steel=3, Glass Fibre Reinforced Polyester = 9 (GRP), Glass Fibre Reinforced Epoxy = 12 (GRE).

Wake probability (0.06 is recommended)

Very large wind farm (increased ambient turbulence)

☐ Large wind farms: Increased ambient turbulence

The model may take an increased ambient turbulence level into account, if the site has many turbines. The effective turbulence model suggest that increased turbulence should be taken into account if there are more than five rows of turbines with more than five turbines in each row.

Geometry settings

☒ WindPRO calculates geometric settings in model

Neighbor turbines

Distance between wind turbines within a row [m]

Distance between wind turbine rows [m]

WindPRO may calculate the number of neighboring turbines. However, when the wind farm has complex geometry, this estimation may fail. Alternative, you may manually specify the number of neighboring turbines (model parameter <= 8).

Cancel Ok

Quarton / Ainslie

For this model there are two suggestions for the parameter setting including the option to user-define them. These options and the parameters are explained in the theoretical section.

Model: Empirical wake added turbulence model proposed by Quarton and Ainslie
Reference: D.C. Quarton and J.F. Ainslie: Turbulence in Wind Turbine Wakes, Wind Engineering - Vol. 14 - No. 1 - pp. 15-23

Description	Value
Global proportional constant (default 4.80)	4,800
Ct exponent (default 0.70)	0,700
lamb exponent (default 0.68)	0,680
Distance ratio exponent (default -0.57)	-0,570

Original constants (table read-only) ▼

Ok Cancel

With the lower menu box it is possible to change between parameter settings.

Empirical turbulence – Dutch TNO laboratory.

Here the following parameters are available. They are explained in theory.

Model: Empirical added turbulence model proposed by the Dutch TNO laboratory
Reference: European Wind Turbine Standards II, ECN-C--99-073, 1999

Description	Value
Global proportional constant (default 1.31)	1,310
Ct exponent (default 0.70)	0,700
lamb exponent (default 0.68)	0,680
Distance ratio exponent (default -0.96)	-0,960

Ok Cancel

DIBt 2004

The parameters available for this German standard are identical to the Sten Frandsen model.

Wake added turbulence

The Wake added turbulence calculation is included in the PARK calculation if the check box with the same name is hatched in the Main tab sheet of the PARK calculation.

The calculation will result in a report page for the entire wind farm and a sheet for each of the turbines.

The common page gives the general calculation parameters and the below table which presents the maximum turbine intensity at 15 m/s and the associated wind direction. This turbulence intensity is a combination of the ambient turbulence and the wake induced turbulence.

Free Wsp: 15,0 m/s							
Manufact.	Type	Power	Diam.	Height	Dir	Wsp	It
		[kW]	[m]	[m]	[deg]	[m/s]	[%]
1	VESTAS	V39	600	39,0	40,5	277	15,0 13,1
2	VESTAS	V39	600	39,0	40,5	261	15,0 13,1
3	VESTAS	V39	600	39,0	40,5	258	15,0 13,1
4	VESTAS	V39	600	39,0	40,5	309	15,0 11,9
5	VESTAS	V39	600	39,0	40,5	283	15,0 12,4
6	VESTAS	V47	660	47,0	40,0	308	15,0 11,8
7	VESTAS	V47	660	47,0	40,0	255	15,0 11,3
8	VESTAS	V47	660	47,0	40,0	255	15,0 11,3

For the individual turbine pages the page can be designed from the Report setup window to contain a number of different results.

The screenshot shows the 'Report setup' window with the following sections:

- Reports:** A list of report components with checkboxes and page numbers. 'WTG Turbulence: Main Result' is selected and highlighted in blue.
- Pages:** A list of pages corresponding to the reports. Page 11 is highlighted.
- Turbulence:** A section with two sub-sections:
 - Include these wind speeds in the report:** A list of wind speeds from 0,5 m/s to 5 m/s. The 15 m/s option is selected.
 - Include these graphs in the report:** A list of graph types. 'Mean ambient turbulence intensity', 'Maximal total turbulence intensity', 'Mean total turbulence intensity', and 'IEC 61400-1:1999 turbulence classes A+B' are selected.
- Wind Speed Reduction: Graph type:** A section with two radio buttons. 'Show as wind rose (at mean wind speed)' is selected.
- Buttons:** 'Result to file', 'Report language' (set to English), 'Use for all calculations', 'Use selected as default', 'Clear default', 'Preview', 'Print', 'Ok', and 'Cancel'.

The turbulence can be calculated for a number of different wind speeds where 15 m/s is the default selection.

Then the graphs and tables can show mean and/or maximal ambient and/or total turbulence. Also the requirements for IEC 61400-1 turbulence classes can be included and thus compared to the calculated turbulence.

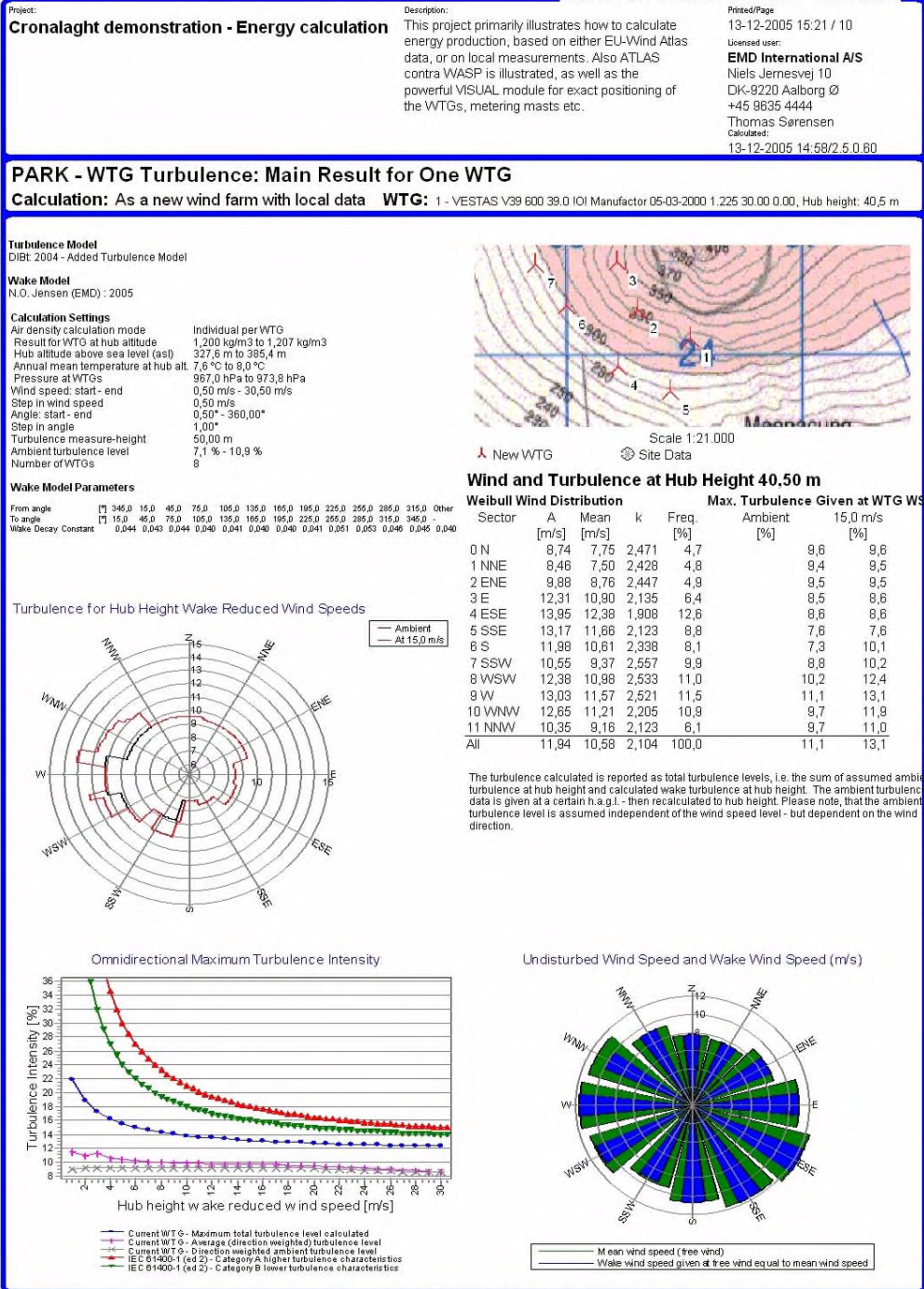
Finally the wind speed reduction can be shown as either a wind rose or as polar line graphs.

The wind speed, turbulence, A and k parameters are presented for each direction on the report page as shown below. The turbulence data is presented also in the form of a diagram. The maximum turbulence,

User Guide

average direction weighted turbulence and the average ambient turbulence is compared to the IEC 61400-1 codes

WindPRO version 2.5.0.60 Dec 2005



Reduced wind speeds inside wind farm

Checking the check box of the same name include the “Reduced wind speeds inside wind farm” calculation.

The only option to choose from the report setup page is the free wind speed at which the wind speed reductions should be calculated.

The report page is shown below and illustrates the wind speed reductions as vectors giving the direction for maximum wind speed reduction and the magnitude of this reduction.

The wind speed reductions can also be printed to a text file or copied to clipboard from the report setup. This is done through this window.

The screenshot shows a software window with a light beige background. It contains two columns of checkboxes for selecting parameters. The first column is titled "Wind speed [m/s]" and lists values from 0,5 to 3,5 in increments of 0,5. The second column is titled "Angle (Clock wise from north) [°]" and lists values from 0,50 to 6,50 in increments of 0,50. Below these columns is a section titled "WTGs" with a list of turbine IDs: 9850, 9852, 9853, 9851, 9854, 11054, 11055, and 11056. At the bottom of the window are three buttons: "Copy to clipboard", "Select all", and "Close".

Wind speed [m/s]	Angle (Clock wise from north) [°]
<input type="checkbox"/> 0,5	<input type="checkbox"/> 0,50
<input type="checkbox"/> 1,0	<input type="checkbox"/> 1,50
<input type="checkbox"/> 1,5	<input type="checkbox"/> 2,50
<input type="checkbox"/> 2,0	<input type="checkbox"/> 3,50
<input type="checkbox"/> 2,5	<input type="checkbox"/> 4,50
<input type="checkbox"/> 3,0	<input type="checkbox"/> 5,50
<input type="checkbox"/> 3,5	<input type="checkbox"/> 6,50

WTGs

- ☐ 9850
- ☐ 9852
- ☐ 9853
- ☐ 9851
- ☐ 9854
- ☐ 11054
- ☐ 11055
- ☐ 11056

Copy to clipboard Select all Close

In this way the wind speed reduction can be analyzed for particular wind speeds wind directions and turbines.

Project: Cronalaght demonstration - Energy calculation	Description: This project primarily illustrates how to calculate energy production, based on either EU-Wind Atlas data, or on local measurements. Also ATLAS contra WASP is illustrated, as well as the powerful VISUAL module for exact positioning of the WTGs, metering masts etc.	Printed/Page: 13-12-2005 15:41 / 1 Licensed user: EMD International A/S Niels Jernesvej 10 DK-9220 Aalborg Ø +45 9635 4444 Thomas Sørensen Calculated: 13-12-2005 15:33/2.5.0.60
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PARK - Wind Speeds Inside Wind Farm: Main Result

Calculation: As a new wind farm with local data

Wake Model N.O. Jensen (EMD) : 2005

Calculation Settings

Air density calculation mode
 Result for WTG at hub altitude
 Hub altitude above sea level (asl)
 Annual mean temperature at hub alt.
 Pressure at WTGs

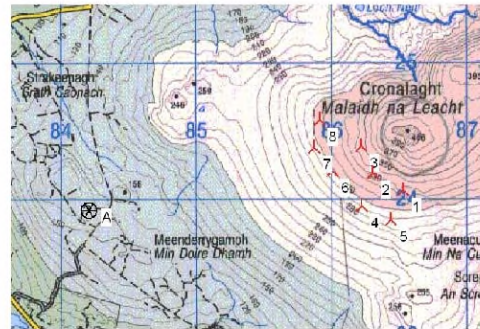
Individual per WTG
 1,200 kg/m³ to 1,207 kg/m³
 327,6 m to 385,4 m
 7,6 °C to 8,0 °C
 967,0 hPa to 973,8 hPa

Wake Model Parameters

From angle	345,0	15,0	45,0	75,0	105,0	135,0	165,0	195,0	225,0	255,0	285,0	315,0	Other
To angle	15,0	45,0	75,0	105,0	135,0	165,0	195,0	225,0	255,0	285,0	315,0	345,0	
Wake Decay Constant	0,044	0,043	0,044	0,040	0,041	0,040	0,040	0,041	0,051	0,053	0,046	0,045	0,040

Wind statistics

IE 30,0 m Cronalaght met mast wws

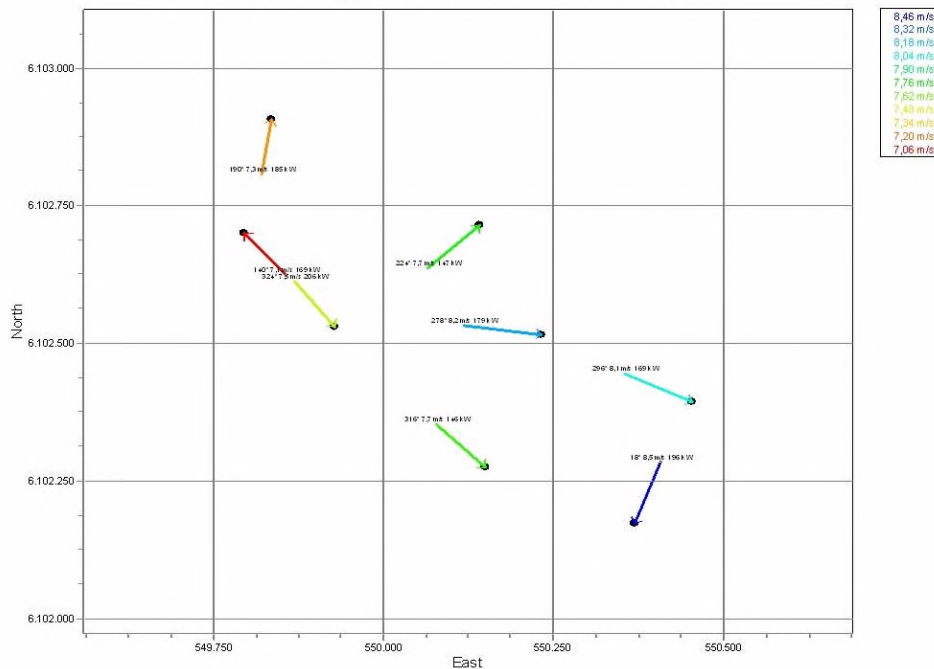


Scale 1:40.000

▲ New WTG

● Site Data

Free Wind Speed: 10,0 m/s
 Maximum Reduced Wind Speed Inside Wind Farm and the Concurrent Direction



WindPRO is developed by EMD International A/S, Niels Jernesvej 10, DK-9220 Aalborg Ø, Tlf. +45 96 35 44 44, Fax +45 96 35 44 46, e-mail: windpro@emd.dk

Park power curve based on PPV model

The Park Performance Verification model is a way to verify the performance of a wind farm by comparing it with concurrent measurements at a nearby meteorological station.

The PPV model establishes the connection there is between wind speed and wind direction at the mast with production output of the wind farm. The result of the PPV calculation is a table like below with production as function of speed and direction

Wind speed [m/s]	Park	WTG: N	NNE	ENE	E	ESE	SSE	S	SSW
[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]
0,5	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1,5	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
2,5	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
3,5	8	25	9	0	0	4	25	6	3
4	65	104	57	14	31	108	113	63	38
4,5	185	231	154	94	183	268	271	158	117
5	348	408	308	244	353	457	457	319	242
5,5	541	621	486	411	548	676	681	507	405
6	760	864	687	598	766	929	934	722	587
6,5	1.015	1.138	928	816	1.022	1.223	1.227	966	803
7	1.298	1.439	1.192	1.057	1.304	1.557	1.557	1.238	1.044
7,5	1.616	1.773	1.487	1.328	1.627	1.926	1.922	1.546	1.314
8	1.959	2.127	1.805	1.627	1.973	2.321	2.315	1.878	1.609
8,5	2.318	2.485	2.141	1.950	2.342	2.728	2.715	2.224	1.920
9	2.683	2.843	2.484	2.290	2.720	3.134	3.114	2.582	2.248
9,5	3.049	3.196	2.830	2.642	3.100	3.527	3.492	2.942	2.585
10	3.399	3.514	3.166	2.992	3.468	3.882	3.840	3.292	2.918
10,5	3.722	3.814	3.483	3.334	3.805	4.185	4.135	3.617	3.242
11	4.008	4.069	3.769	3.654	4.106	4.430	4.385	3.907	3.542
11,5	4.249	4.289	4.024	3.948	4.347	4.616	4.575	4.158	3.817
12	4.446	4.465	4.242	4.195	4.544	4.749	4.720	4.362	4.058

In order to make a PPV model in WindPRO there must be a site data object for WasP calculation on the location of the meteo mast. It is not necessary that this site data object hold a relevant wind statistic as the site data object for the PARK calculation will be used.

The site data object and the hub height is chosen in the PARK calculation set up (below)

The result is obtained through “Result to file” in the Report setup where “Park power curve” is chosen.

Annex A: Case Study – Horns Rev Offshore Wind Farm

This annex is mainly based on a paper presented at the EWEA 2006 conference in Athens, Greece.

Recalibrating Wind Turbine Wake Model Parameters

– Validating the Wake Model Performance for Large Offshore Wind Farms

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Summary

As part of the Danish PSO sponsored project '*The Necessary Distance between Large Wind Farms at Sea*' EMD International A/S has implemented a number of wake models in the WindPRO software. In this paper we report the preliminary results of a case study on Horns Rev offshore wind farm, where the actual observed wake losses are compared with calculations using the implemented wake models. The wake loss can be analyzed by sector and wind speed, which in the future allow for improved parameterization of the models. This case study indicated that the traditional N.O. Jensen wake model is more precise at predicting the observed wake loss than the other tested wake models, at least when the current default parameters are used.

Introduction

The aim of the analysis is to verify/improve existing wind turbine wake models through model parameter adjustments, so that they can be utilized in large offshore wind farms. The analysis includes parameter sensitivity studies on three different wake models, the N.O. Jensen model [1], the Ainslie model (Eddy Viscosity) [2] and the G.C.Larsen model (Prandtl BL-equations) [3]. The performance of each of the models is compared to data based on the performance of offshore wind farms. The focus of the analysis is – primarily – to predict energy output for the wind farm as well as for single wind turbines in the farm. The secondary objective is to predict the mean wind speeds and turbulence in the wakes.

Motivation

Most wind turbine wake models - used in wind farm evaluations today - are based on the single wake flow downstream of a wind turbine. The flow from each of the single wakes is then added into a combined effect – using a simple empiric combination model. This model is normally a 'quadratic wind speed-deficit model'. In connection to the Danish research project, '*The Necessary Distance between Large Wind Farms at Sea*', EMD has implemented two alternative wake models as alternative to the widely used N.O. Jensen model [4]. All three wake models can be used for energy calculations, mean wind field calculations and with turbulence-calculations. The models are varying in complexity from a simple empiric engineering model to an axi-symmetric CFD-model. All the models still lack a structured validation and calibration for use on large offshore farms, just as the used wake combination model is not necessarily applicable for this purpose.

Current Progress (February 2005)

A preliminary wake study has been completed for one offshore site and a number of other wind farms are being prepared for study. Model parameters have through previous studies and literature been suggested and these are tested against this first case. The study will proceed with parameter adjustments on this first site and other wind farms in order to align these models to a correct prediction.

Expected results

The offshore measurements at the demonstration wind farms located in Danish waters contain a powerful potential for improving the existing wake models. The use of validated wake models gives a high degree of certainty for project developers running analyzes on large offshore wind farms. As the project is based on re-calibration of offshore specific

parameters for existing and already validated models the results will - on a fairly short term - be able to improve the estimates and decrease the uncertainties for these models.

The project

The project has partly been funded by a grant from the Danish public service obligation (PSO) R&D program. The project runs from primo 2005 to ultimo 2006.

Test case Horns Rev.

The Horns Rev offshore wind farm was erected end 2002 and consist of 80 Vestas V80-2.0MW wind turbines. The wind farm is located 13 km from the west coast and the turbines are placed with a spacing of 7 rotor diameters. The first years of operation the turbines were suffering from poor availability, but this have been improved and for 2005 the availability was 95% according to the operator (Elsam) [5].

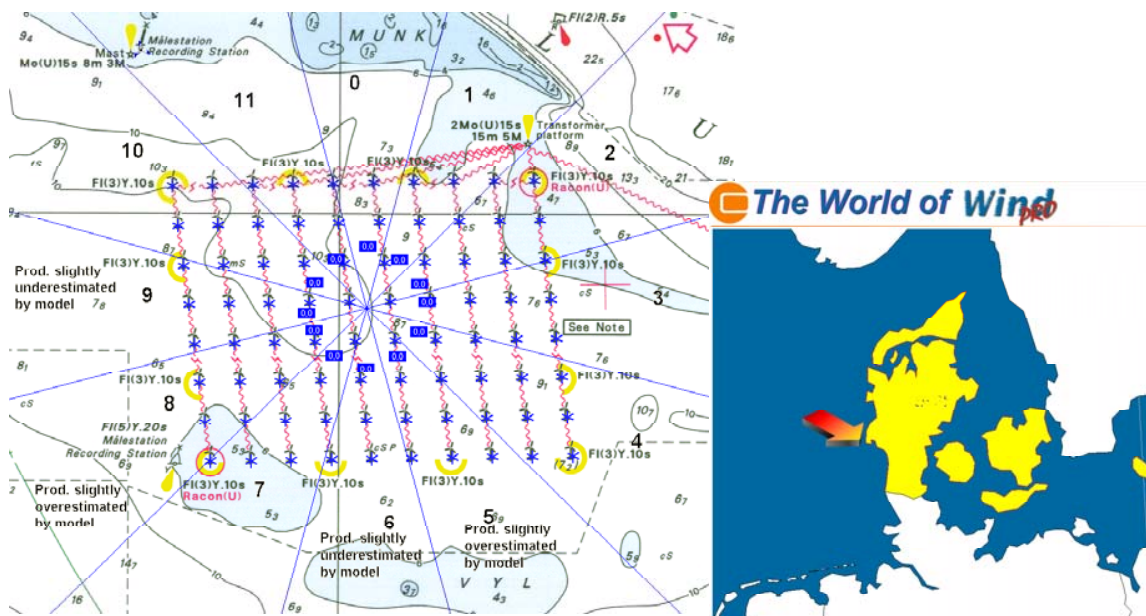


Figure 1. Map of the Horns Rev wind farm with sector numbering, showing the 12 direction sectors in which data are grouped in the analyses.

Measured wake loss.

The operator Elsam has (through Elsam Engineering) developed a SCADAVIEW system [5] that is able to produce the deficiency in production for the wind farm compared to a free standing turbine as a function of wind speed and wind direction. The free standing turbine is the corner turbine which is most exposed to the wind direction in question. In so far as that turbine is operating correctly this system provides measurements of actual wake losses.

Test environment.

EMD has created a test system where the measured wake loss can be compared to the calculated wake loss. Park production calculations are run in WindPRO with the model and parameter settings wanted and the result is exported as a Park power curve (PPC). The PPC tells what the cumulative effective power curve has been for the wind farm as a whole. The difference from a simple multiplication of the power curve with the number of turbines and the PPC is the wake loss. In this way measured and calculated wake loss can be compared as a function of wind speed and direction. In addition by employing a representative Weibull distribution these individual wake losses can be converted to the combined wake loss. The total measured park efficiency is 87,6%, which corresponds to a wake loss of 12,4%.

Preliminary test runs

The wake models mentioned in the introduction and further described in the WindPRO manual [4] have been tested in this environment with a few tests of different parameter settings. The test settings were:

N.O. Jensen (old) [1]: This is the standard model used in previous versions of WindPRO and by WAsP's Park model. The only parameter which can be adjusted is the Wake Decay Constant (WDC), which has been tested for WDC = 0.04 and 0.075, which are recommended settings for offshore and onshore conditions respectively.

N.O. Jensen (2005): This is a modification of the old N.O. Jensen model to accommodate a new system for addition of wakes and includes optionally wake induced turbulence calculation [4]. Only WDC = 0,04 is tested.

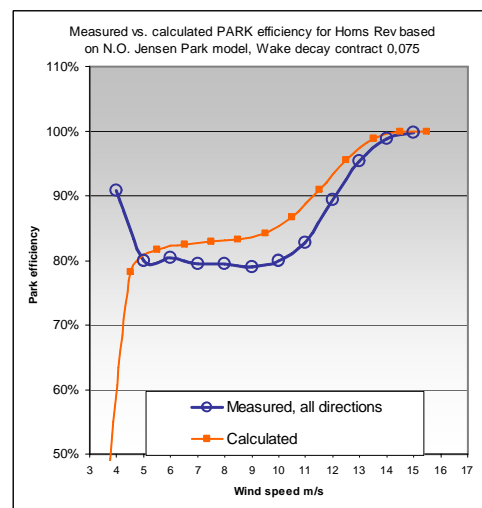
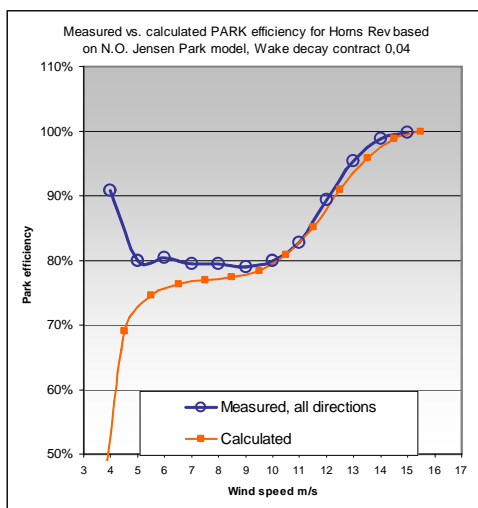
EWTS II (1999): This is a new model in WindPRO suggested by G.C.Larsen (1999) [3]. WDC = 0,04 is tested. Standard parameters as described in the WindPRO manual are used for a first order calculation.

Eddy Viscosity model (1986): This model was suggested by J.F. Ainslie (1986) [2] and is new in WindPRO. This one has several parameters to adjust. In this test standard settings for WindPRO as described in the manual are used except for the constant K1, which is tested for the values 0,015 (standard) and 0,025.

As a special test suggested by some researchers the roughness inside the wind farm has been increased to $z_0=0,05\text{m}$ to reflect the roughness change induced by the wind farm itself.

Results

As the below figures show, the N.O. Jensen with offshore WDC = 0.04 is the most accurate model to predict the wake losses. The old version is slightly tighter to the measured values than the new version (N.O.Jensen 2005 as implemented in WindPRO 2.5 in addition to the old to make Wake turbulence calculation optional). Increasing the WDC to 0,075 seems to be a poor idea. Both EWTS II and the Eddy Viscosity model seem to under predict the wake loss and therefore over predict the production.



Case Study – Horns Rev

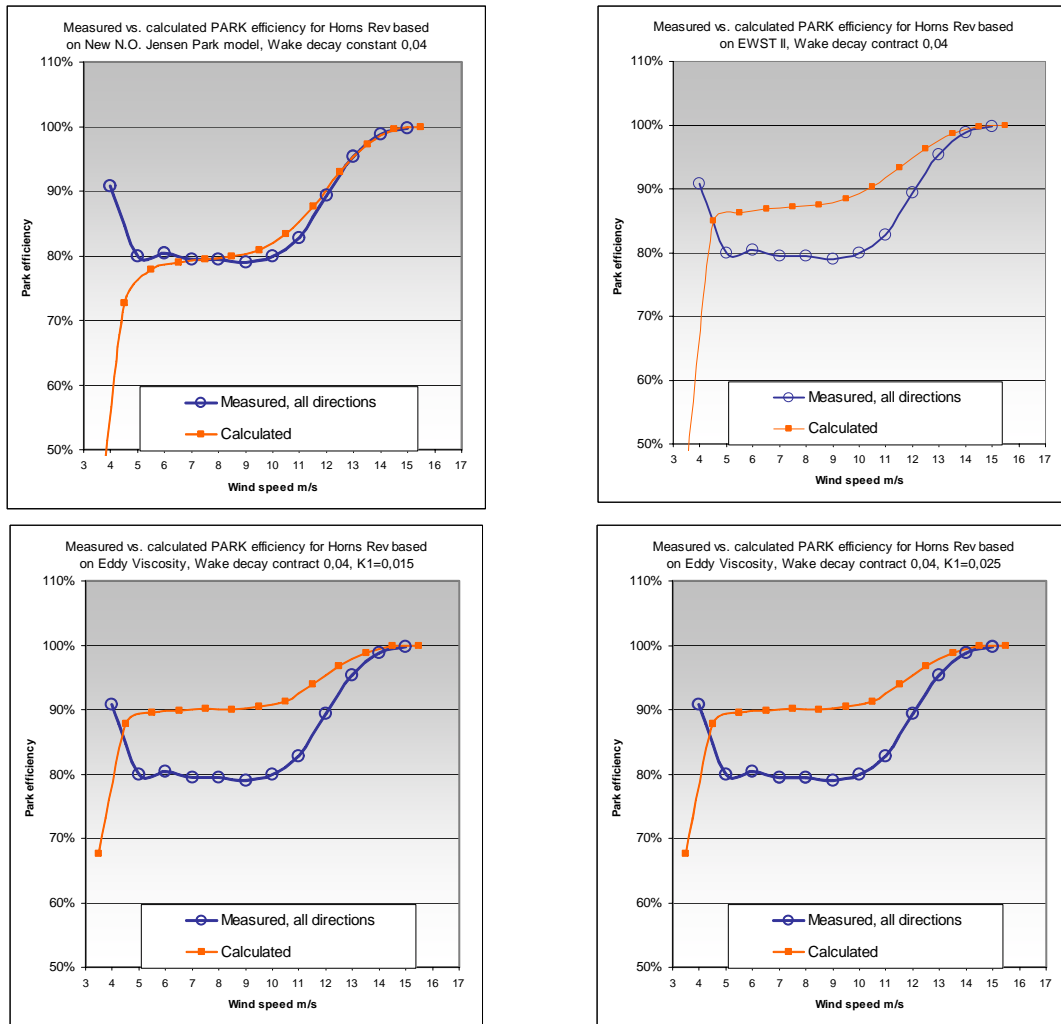


Figure 2. Preliminary tests of four available methods with standard conditions or limited parameter variation. The graphs plot the measured and calculated park efficiency for all sectors as a function of wind speed.

Even for the best predicting model there is variation as to how well each direction is predicted. Below are shown two examples of a good and less good prediction at two different sectors.

Case Study – Horns Rev

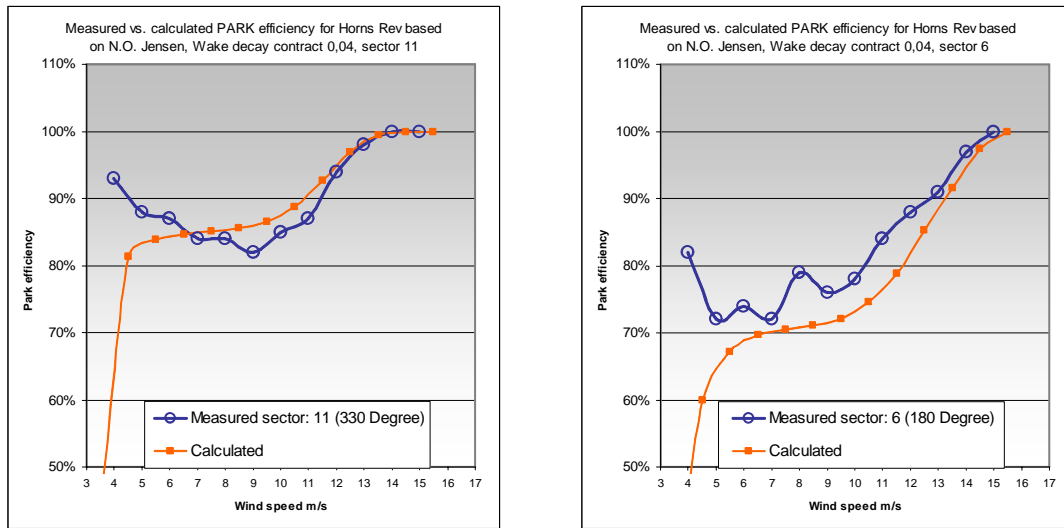


Figure 3. Even though the total fit between measured and calculated park efficiency of the old N.O. Jensen model is good, the individual sector fits can be less accurate as the two examples above illustrate.

The total difference in measured and calculated wake losses is illustrated in figure 4. Negative values are due to models that under predict the wake loss and therefore calculate too high a production. It is clear that the old N.O. Jensen for this case seems superior as long as a reasonable WDC is used.

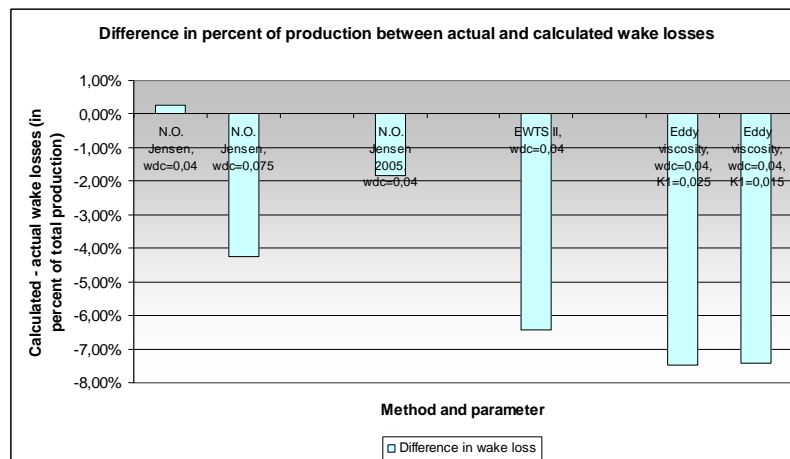


Figure 4. The ability of the models to accurately predict the measured wake loss is illustrated by this figure. A negative value of e.g. 2% means that the calculation model calculate the wake loss 2% of total production less than actually observed and therefore total production 2% higher. Most of the models under predict the wake loss, except for the old standard N.O. Jensen model that apparently is able to accurately predict the wake losses.

If an internal roughness of 0,05m is introduced inside the wind farm, the wake losses remain the same but since the base calculated production is reduced the wake loss deficit can be attenuated. This is illustrated in figure 5, which apparently

improves the performance of the poorly performing EWTS II and eddy Viscosity, but offsets the otherwise well performing N.O. Jensen model. While an internal roughness seems to be a good idea at other locations it is apparently not appropriate on this location.

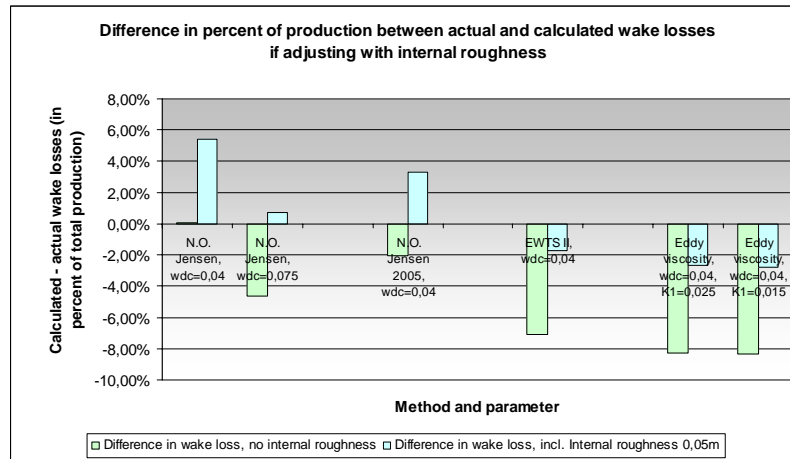


Figure 5. Introducing an internal roughness area of $z_0=0,05\text{m}$ inside the wind farm lower production and can thus compensate for the lack of predicted wake loss. However for well predicting models like N.O. Jensen this does not necessarily improve the prediction.

Conclusion

The N.O.Jensen model with $WDC=0.04$ seem to predict the measured array losses for the Horns Rev wind farm very accurate. Other models under predict losses typically around 6 to 8% of total production and thereby overestimate production. For other large wind farms tested, but not reported in this paper, it seems that even the most conservative of the models, the old N.O.Jensen, under predict array losses. The reason for correct prediction of Horns Rev might be the very open offshore location with high mean wind speed and real open sea stability conditions.

Future work

The plan is to set up a few other cases in order to be able to calibrate the different models and define the proper procedures and parameters choice for the models used.

References

- [1] Katić, Højstrup J, & Jensen N.O. A Simple Model for Cluster Efficiency, *European Wind Energy Association, Conference and Exhibition*, 7-9 October 1986, Rome, Italy.
- [2] Ainslie, J.F: Calculating the flowfield in the wake of wind turbines, *Journal of Wind Engineering and Industrial Aerodynamics* 1988, **27**: 213-224.
- [3] Larsen G.C, Højstrup J, Madsen HA. Wind Fields in Wakes, *EUWEC '96*, Gothenburg, 1996.
- [4] Nielsen P, et.al., *The WindPRO manual edition 2.5*, EMD International A/S, 2006.
- [5] Leo E. Jensen, Personal communication on the SCADA results from Horns Rev Wind farm, Elsam Engineering 2006.

Annex B: Case Study – Wake Added Turbulence at Nørrekær Enge

This annex is mainly based on a paper presented at the EWEA 2006 conference in Athens, Greece.

Evaluating Models for Wind Turbine Wake Added Turbulence – Sensitivity Study of the Models and Case Study

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Summary

A range of turbulence models for wake added turbulence has been implemented in the WindPRO software. These models have been parameterized according to recommendations from the researchers who published or revised the models or the guidelines from which the model originate. The authors of this paper are in the process of validating these turbulence models by use of case studies. This paper presents the preliminary results from one such case study: The Nørrekær Enge wind farm in Denmark. Using two meteorological masts in and on the perimeter of the wind farm the ambient turbulence at both places has been measured. The difference is the wake added turbulence. An initial setup of 13 different combinations of turbulence and wake models has been tested against these measurements. The tests reveal a varying degree of success, both among the model configurations, but also among the direction sectors investigated. They highlight the importance of choosing a proper set of parameters, but also that test cases are highly sensitive to error.

Introduction

Turbines operating in wakes are subjected to significant higher structural loading than turbines operating in the free wind. Appropriate turbulence calculations should be made before selecting the proper turbine design class when having clusters of turbines. In this study, the wake added turbulence has been calculated using three different wake models and seven different turbulence models. These models are typically very different in detailing level – and possible also in accuracy. The models range is from simple engineering models to the more advanced computational fluid dynamic (CFD) models. The CFD-models are typically also very demanding in terms of calculation time.

Turbulence Models and Wake Models Included in the Analysis

In the analysis the following wake added turbulence models have been implemented and tested: Danish Recommendation: 1992, Eddy Viscosity: 2003 (B. Lange), Quarton:1996 (D.C. Quarton & J.F. Ainslie), Dutch TNO Laboratory, G.C.Larsen: 1998 (EWTS II), S. Frandsen: 1999 (Efficient turbulence model) and the DIBt Richtlinie: 2004. The turbulence model must be used in connection with a wake (wind field) model. In the analysis, the following wake models are included: PARK model: 1996 (N.O. Jensen), Eddy viscosity model: 1988 (J.F. Ainslie), G.C. Larsen: 1998 (European Wind Turbine Standards II). A description of these models including references can be found in the WindPRO manual [1].

Sensitivity Studies

The turbulence model parameters will be subjected to a sensitivity analysis to test the performance of the models under various environmental conditions. The performance of the models will then be compared.

Case Study – Wake Added Turbulence at Nørrekær Enge

Case Studies

The ambient turbulence level from measurements in a number of international wind farms will be compared with calculated predictions of ambient + wake added turbulence. The performance of the models will be compared.

Progress (February 2006)

The combinations of wake and turbulence models have been tested on the wind farm Nørrekær Enge in Denmark. At this stage the models have been using standard settings with the intention of fine tuning these with a sensitivity study. The preliminary results are reported below.

The case: Nørrekær Enge

Nørrekær Enge is a wind farm in the Northern part of Denmark that was erected in 1988-90. When it was erected it was one of the largest of its kind with 36 130 kW and 42 300 kW Nordtank wind turbines. The utility Elsam operates the wind farm and the production is well documented. The turbines are located as illustrated in figure 1 in two groups with an internal spacing of 6-7 times rotor diameter. From 1991 to 1993 two metering masts have collected wind speed and turbulence readings at hub height (31 m). Their location is shown in figure 1. One is located on the southern edge of the wind farm and is thus undisturbed from sector 4 to 8. The second is located inside the wind farm near the east end and is influenced from all directions. With a distance of only 1800 m between the masts in a non complex landscape it is reasonable to assume that the ambient turbulence for the concurrent period is similar. Any additional turbulence at mast 2 from sector 4 to 8 will be wake added turbulence.

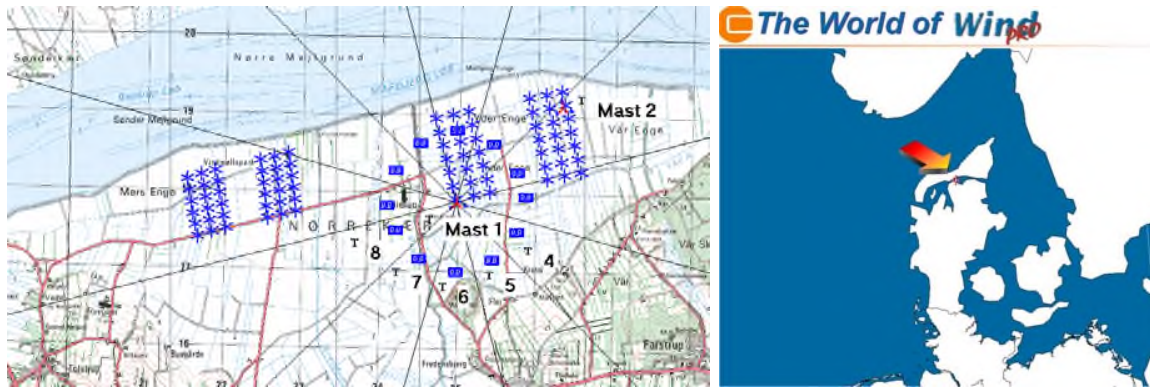


Figure 1. Outline of the test case Nørrekær Enge. The blue symbols are the wind farm, while the red symbols mark the two metering masts. Sector 4-8 are outlined at mast 1.

The measurements

A section of the measurements is isolated where 1) there are concurrent healthy data on both masts and 2) all turbines are in operation. This leaves 24000 measurement points. Turbulence intensity (TI) is calculated from 10 minute mean wind speed readings and standard deviation on same. The TI readings are grouped so mean wind speed and standard deviation is obtained for every 1 m/s wind speed bin and 12 direction bins. From this, representative turbulence is calculated as recommended in IEC 61400 vs. 2 and vs. 3, that is respectively as $\text{mean} + 1 \cdot \text{std.dev of TI}$ and as $\text{mean} + 1.28 \cdot \text{std.dev of TI}$.

Observations from sector 4 to 8 are extracted for the typical wind speeds of 9.5, 14.5 and 19.5 m/s.

Calculation of turbulence

The calculation of wake added turbulence is an integral part of a standard energy production calculation using the WindPRO module PARK. A standard setup for an energy calculation is made using an orographic and roughness description and the wind atlas Danmark 92, which has in previous studies been shown to predict the wind farm production

Case Study – Wake Added Turbulence at Nørrekær Enge

well. Wake models and turbulence models from the list mentioned above is chosen with the appropriate parameter settings. As for ambient turbulence, the readings from mast 1, which is undisturbed in the investigated sectors is imported and used for each sector. This is also used to calculate the wake decay constant for the wake models. The turbulence is calculated for a virtual turbine at the location of mast 2.

The following combinations and parameter settings were tested. “0” means no parameter setting available. Standard parameters are the default parameters used in WindPRO.

Configuration	turbulence model	Parameters	Wake model	Parameters
1	EWTS II	0	N.O. Jensen	0
2	EWTS II	0	EWTS II	1.order
3	EWTS II	0	Eddy vis.	Standard
4	Danish recommendations	Gridded layout	N.O. Jensen, 2005	0
5	Steen Frandsen	Wohler =3, wake prop.=0,06	N.O. Jensen, 2005	0
6	Steen Frandsen	Wohler =9, wake prop.=0,06	N.O. Jensen, 2005	0
7	Steen Frandsen	Wohler =12, wake prop.=0,06	N.O. Jensen, 2005	0
8	Steen Frandsen	Wohl =9, wake prop=0,06, large w	N.O. Jensen, 2005	0
9	Quarton	Standard	Eddy Viscosity	Standard
10	B Lange	Standard	Eddy Viscosity	Standard
11	Dutch TNO	Standard	N.O. Jensen, 2005	Standard
12	Dutch TNO	Standard	EWTS II	Standard
13	DIBT	Wohler =3, wake prop.=0,06	N.O. Jensen, 2005	Standard

Results

The observed representative (vs.2) TI for mast 1 and mast 2 is shown in figure 2. In some sectors there are no measurements of the higher wind speeds at the mast. Turbulence is higher at mast 2 due to turbulence from the wakes.

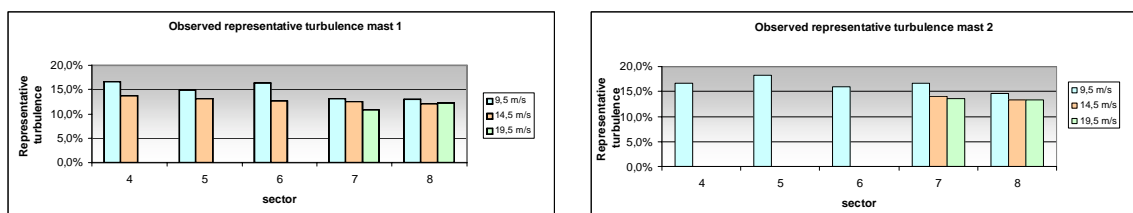
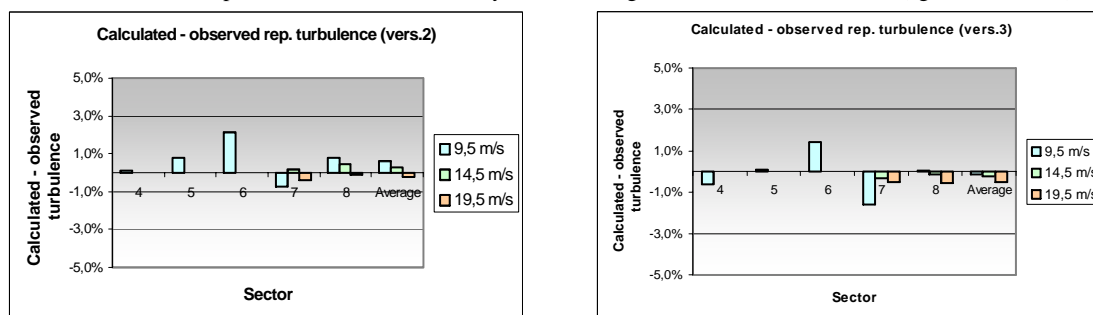


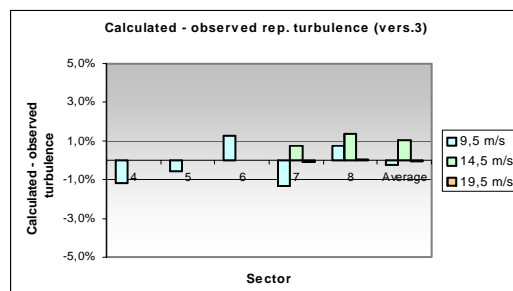
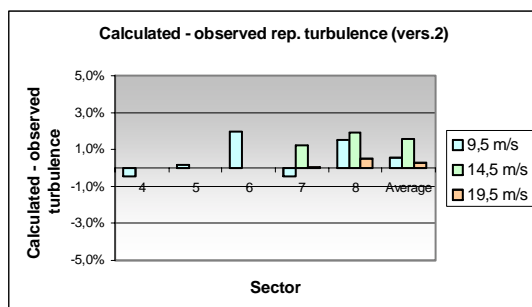
Figure 2. Observed representative (vs.2) turbulence intensity at mast 1 (reference) and mast 2 for three wind speeds and five sectors.

For each of the calculations the difference in calculated TI to the observed TI is plotted for a few representative configurations below. For sector 4 to 6 this is only possible for wind speed at 9.5 m/s. A positive difference of 1% means that the calculation model predicts a turbulence intensity that is 1% higher than observed at mast 2 (eg. 15% vs. 14%).

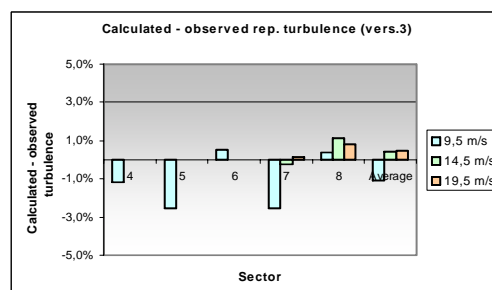
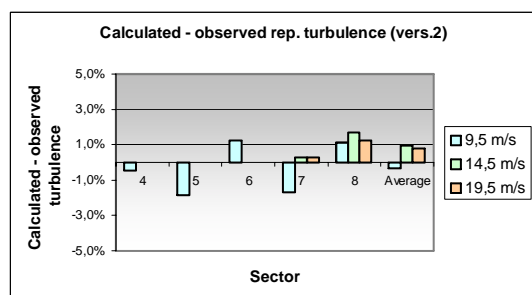


Configuration	turbulence model	Parameters	Wake model	Parameters
2	EWTS II	0	EWTS II	1.order

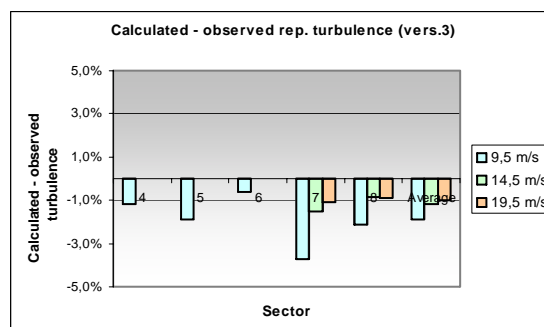
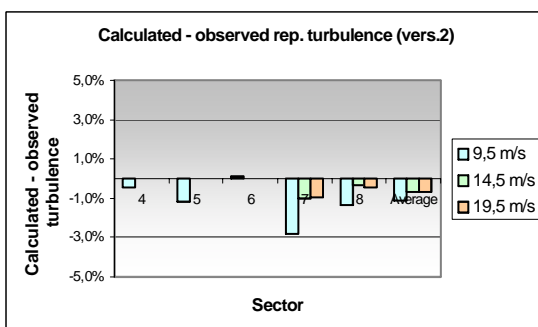
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Configuration	turbulence model	Parameters	Wake model	Parameters
4	Danish recommendations	Gridded layout	N.O. Jensen, 2005	0



Configuration	turbulence model	Parameters	Wake model	Parameters
5	Steen Frandsen	Wohler =3, wake prop.=0,06	N.O. Jensen, 2005	0



Configuration	turbulence model	Parameters	Wake model	Parameters
9	Quarton	Standard	Eddy Viscosity	Standard

Case Study – Wake Added Turbulence at Nørrekær Enge

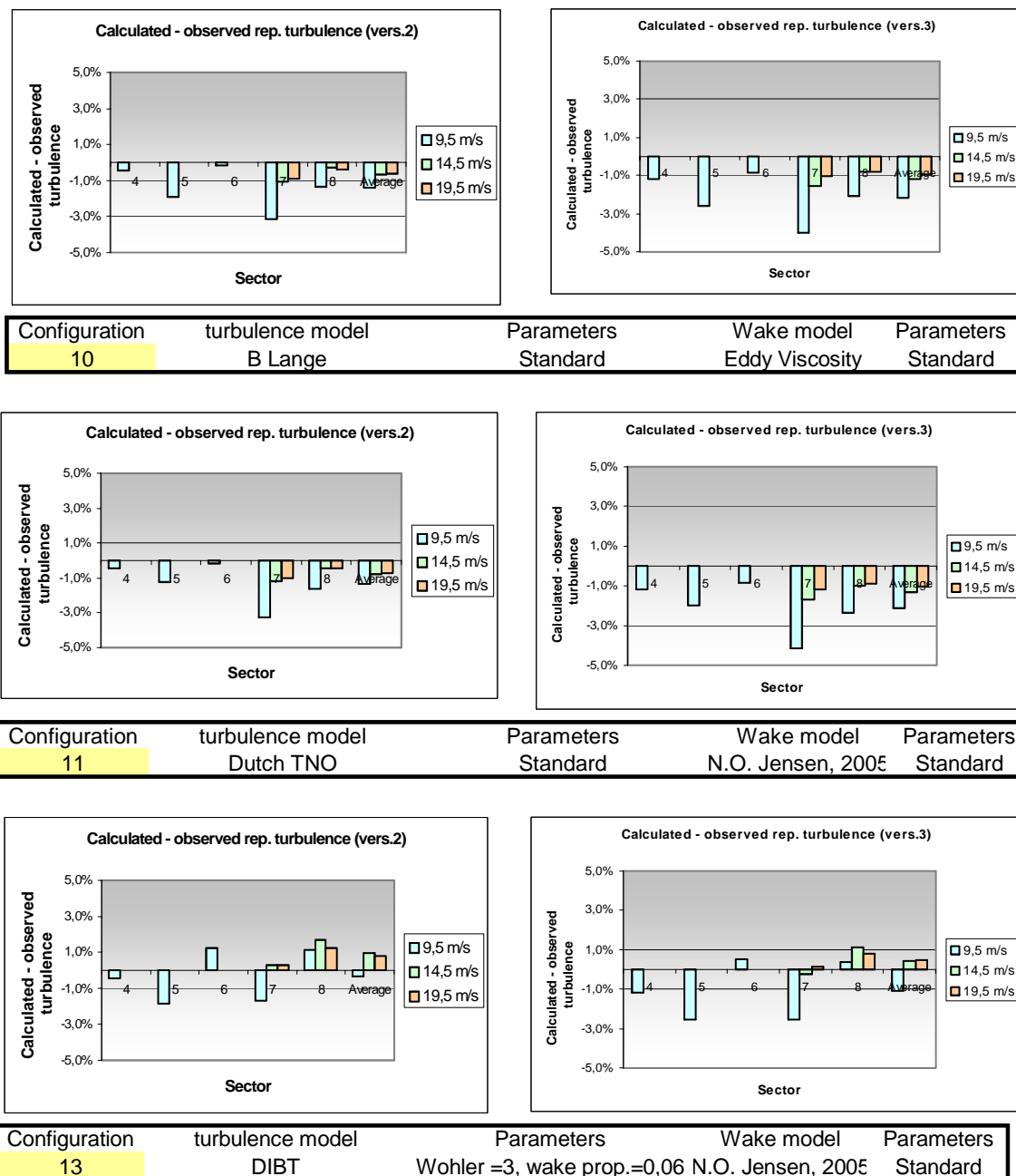


Figure 3. Difference in calculated TI to the observed TI for some of the calculation configurations.

For some of the configurations the calculated wake added turbulence is closer to the observed representative turbulence as calculated according to version 2, while others are closer with version 3. It can also be seen that the precision varies from sector to sector.

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The calculated – observed turbulence results at 9.5 m/s in each sector are illustrated in figure 4 for each of the 13 tested configurations. The number in x axis refers to a turbulence model configuration from the table above. The average figure is an average of all three wind speeds and all sectors.

Where all the models agree in sector 4 where there is no significant wake influence at mast 2, the variation from model to model gets quite significant in the more disturbed sectors. Sector 8 most notably is calculated very differently with the Steen Frandsen turbulence model with a Wöhler curve exponent of 12 (config. nr.7), than with the Dutch TNO turbulence model (config. nr.11 and 12).

The parallel shifts between the sectors could indicate systematic errors in the observed turbulence intensity.

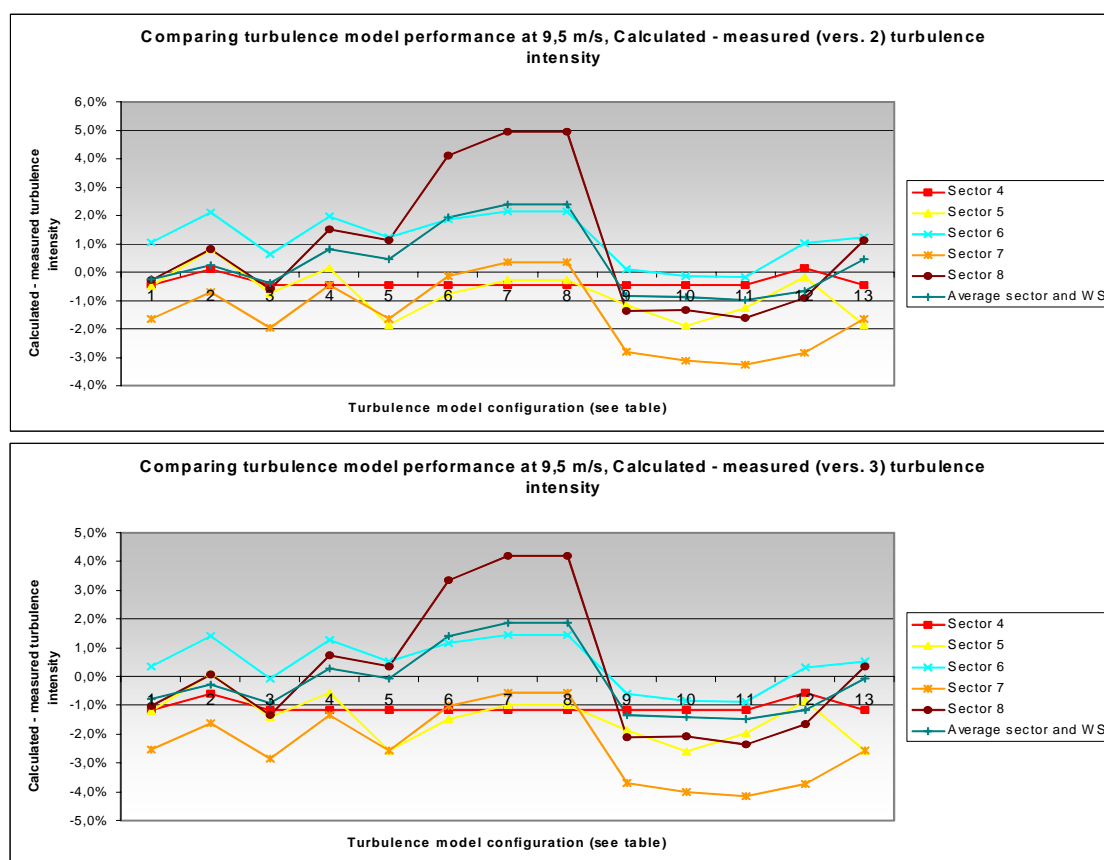


Figure 4. Difference in calculated TI to the observed TI across the configurations tested (please refer to the table) for each sector. The average is an average of all sectors and all three wind speeds.

Conclusion

Testing the different wake added turbulence models and comparing the results with measured data, gives an overview of the model performance in various conditions. This case study begins this work. So far, the following observations based on this example can be made:

Some turbulence models clearly need a parameter calibration, or the user must at least be careful with the parameter settings. The precision varies from model to model, not necessarily with the most advanced being the most precise models. A case study is very sensitive to the precision of measured turbulence. If the ambient turbulence at the test site is different

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from the reference site it offsets the results. If a model should be pointed out from this preliminary study then the EWTS II seem to perform better than average.

References

1. Nielsen P, et.al., *The WindPRO manual edition 2.5*, EMD International A/S, 2006.